AN EVALUATION, USING BEHAVIORAL OBJECTIVES, OF TEST 4, "ABILITY TO DO QUANTITATIVE THINKING," FROM THE IOWA TESTS OF EDUCATIONAL DEVELOPMENT

A Field Report

Presented to

The Graduate Division

Drake University

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Education

---

by

Herman L. Richtsmeier

January 1969

AN EVALUATION, USING BEHAVIORAL OBJECTIVES, OF TEST 4, "ABILITY TO DO QUANTITATIVE THINKING," FROM THE IOWA TESTS OF EDUCATIONAL DEVELOPMENT

by

Herman L. Richtsmeier

Approved by Committee:

Raymond A Hock
Chairman

B E Gilliam

Dean of the Graduate Division

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

THE PROBLEM AND PROCEDURE USED

Many changes have taken place in the last ten years in the teaching of mathematics. Certainly those who can personally compare the present with the past of ten or fifteen years ago realize that a transformation of great magnitude has taken place in the field of mathematics education over the intervening years.[1]

Some of the specific changes that have taken place were stated by Gibbs in an article "The Math Curriculum Changes." Those changes are:

> Increased emphasis is placed on the logical structure of mathematics in order to give students a more adequate foundation for the application of mathematics to the unpredictable demands of future technology. Certain skills developed in the past by extensive drill with a limited range of problem types are left to computers or on-the-job training.
>
> The theory of sets is presented as a more systematic and convenient frame of reference and notation for dealing with a wider variety of kinds and domains of numbers and geometric concepts.
>
> Greater efforts are made to relate mathematics to other disciplines, especially grammar and science, in view of the role of mathematics as the language of science.
>
> Greater emphasis is placed on learning by reading because of the growing technological demand that we train students who will be more self-reliant in the lifelong process of education.
>
> Earlier differentiation between the predominantly

---

[1]Francis J. Mueller, "The Revolution at Sputnik-Plus-Ten," The Mathematics Teacher, LX (November 1967), 698.

> academic and the predominantly mechanical-minded student has been demanded by modern trends toward specialization. It has been facilitated by improvement in the validity and reliability of testing and guidance procedures.
>
> Changes in sequence, in which certain mathematical topics can most efficiently and eagerly be learned during a 12-year school career, have been indicated by several independent classroom and school system experiments.
>
> The increasing mobility of the American family and student requires that training in mathematics be as nearly uniform as state and national authorities can agree upon, within the vocational specializations indicated above.[1]

These and many other changes are incorporated into modern mathematics education today. Examples of curricular changes are the merger of plane and solid geometry into a one-year course; offering one-semester courses in analytic geometry or probability and statistics; and adoption of new courses for students needing a more practical and less rigorous brand of mathematics.

A great deal of research has been done comparing "modern" and "traditional" mathematics programs. The studies reported to date point to the fact that modern programs in mathematics are as effective as traditional programs in developing traditional mathematics skills.[2] The question still remains, however, as to whether the

---

[1] Sam M. Gibbs, "New Math Curriculum Changes," The Texas Outlook, IIL (April 1964), 34-35.

[2] Holland Payne, "What About Modern Programs in Mathematics," The Mathematics Teacher, LVIII (May 1965), 424.

traditional standardized achievement test is testing the mathematics being taught today. It would be very difficult, if not impossible, to compare what is being taught in mathematics today with all standardized achievement tests that are being used in secondary schools at this time. For this reason this study is limited to one achievement test, The Iowa Tests of Educational Development. Further, because the major concern is mathematics, only the section of this test battery that pertains to mathematics will be evaluated.

I. THE PROBLEM

Statement of the problem. It was the purpose of this study to evaluate Test 4, "Ability to do Quantitative Thinking" from the Iowa Tests of Educational Development in terms of the objectives of modern mathematics programs being taught at this time. This involved two steps: (1) The selection of objectives that cover the mathematics being taught in the secondary schools; (2) Comparing these objectives to Test 4 from the Iowa Tests of Educational Development.

Importance of the study. The need to evaluate the Iowa Tests of Educational Development was pointed out by Jack R. Williams, mathematics consultant for the Department of Public Instruction for the state of Iowa. He stated

that one of the two problems of present importance is: "How well do the Iowa Basic Skills Tests and the Iowa Tests of Educational Development test what is being taught in modern school mathematics programs?"[1]

The Iowa Tests of Educational Development are widely used throughout Iowa and the United States. In Iowa, four-hundred eighty one schools gave this test battery during 1967.[2] These schools are using this test as part of their total testing program and in many cases this may be the only achievement battery given.

This writer assumes that the schools using the Iowa Tests of Educational Development are using it for purposes similar to those stated by the authors of the tests. The first purpose is to enable teachers and counselors to keep themselves more intimately and reliably acquainted with the educational development of each high school pupil. Such knowledge will make it easier to adapt instruction and guidance to each pupil's peculiar and changing needs.

The second major purpose is to provide the school administrator with a more dependable and objective basis for evaluating the total educational offering of the school.

---

[1] Statement by Jack R. Williams, personal letter (November, 1967).

[2] Statement by Leonard S. Feldt, personal letter (October, 1968).

The testing program will point up any need for curriculum revision that may exist.[1]

If this testing program is going to be used with the second purpose in mind, then this test should be designed so that it does test for the objectives of mathematics today and not some outdated content or objectives. It would be unfortunate for a teacher or curriculum committee to develop a new course of study or a new basic textbook only to find that the testing program points up the fact that the old curriculum seems to be better. In reality, of course, the real problem might be that the testing program is not up-to-date or does not test for those objectives of a good modern program.

The real problem to be solved is that of the validity of the test. As stated by Lindvall: "As with the use of any procedure, the first quality to be considered is how valid the test will be for the intended purpose."[2] The importance of validity is also pointed out by Ahmann and Glock: "Validity is clearly the most important characteristic of an evaluative instrument. No matter what other

---

[1] *How to Use the Test Results of the Iowa Tests of Educational Development, A Manual for Teachers and Counselors* (State University of Iowa, 1967), p. 7.

[2] C. M. Lindvall, *Testing and Evaluation; an Introduction* (Harcourt, Brace, and World, Inc., 1961), p. 186.

characteristics an instrument may possess, if it is not valid to an adequate degree it is of no value whatsoever."[1]

These statements indicate the great importance that validity has in testing. But as the above statement says, the intended purpose must be kept in mind. At the same time, validity means different things to different people. Validity as described by Lein is:

> Validity refers to the care that is taken to include, in a test, items of prime importance and exclude items of trivial nature; validity refers to the degree to which an instrument parallels the material which has been taught and the way in which it has been taught; validity refers to the degree to which an observational tool provides for objective appraisal of that which is observed; validity refers to the specificity of results obtained by means of a measuring device.[2]

Gronlund offers some ideas to keep in mind which he calls cautions:

> Validity pertains to the results of a test and not the instrument itself, or more specifically, of the interpretation to be made from the results. Validity is a matter of degree. It does not exist on an all-or-none basis, consequently we should avoid thinking of evaluation results as valid or invalid. Validity is always specific to some particular use. It should never be considered a general quality.[3]

---

[1] J. Stanley Ahmann, and Marvin D. Glock, Evaluating Pupil Growth (Allyn and Bacon, Inc., 1963), p. 292.

[2] Arnold J. Lien, Measurement and Evaluation of Learning; a Handbook for Teachers (C. Brown Company, 1967), p. 29.

[3] Norman E. Gronlund, Measurement and Evaluation in Teaching (MacMillan Company, 1965), pp. 60-61.

This last point is of most importance in this study. The use of the test does determine the type of validity that is of importance for a particular situation. Four types of validity have been identified and are now commonly used in educational and psychological measurement. They are: content validity, predictive validity, concurrent validity, and construct validity.[1] Many writers put concurrent validity and predictive validity under one title of criterion-related validity. The reason for this will be explained after each of the four types have been defined more completely.

One of these types, construct validity, is ordinarily studied when the tester wishes to increase his understanding of the psychological qualities being measured by the test.[2] Studies of construct validity basically are attempts to evaluate the theory underlying the test. In establishing construct validity, the following steps are carried out. First, the investigator sets up hypothesis about the test he is using. Second, he collects data to test his hypothesis. Finally, in light of the data collected, he makes an inference as to whether or not his data can explain the students' behavior on the test.

---

[1] Ibid., p. 61.

[2] David A. Payne, Educational and Psychological Measurement (Blaisdell Publishing Company, 1967), p. 79.

Another way of showing construct validity is to correlate a new test with other tests. Tests of a certain type should correlate higher with other tests of the same type than they do with tests of another variety.[1]

The term criterion-related validity is demonstrated by comparing the test scores with one or more external variables considered to provide a direct measure of the characteristic or behavior in question.[2] If the test is used to predict some future performance, then predictive validity is the term used. The amount of predictive validity can be determined by comparing test scores with another measure of performance obtained at a later date. If the test performance is compared with some other current performance, then the term applied is concurrent validity. The amount of concurrent validity is determined by comparing test scores with another measure of performance obtained at approximately the same time.[3] Both items, then, are criterion-related but it depends upon the use of the results whether it would be labled as concurrent or predictive validity.

---

[1] N. M. Downie, Fundamentals of Measurement (Oxford University Press, 1967), p. 95.

[2] John W. French, William B. Michael and others, Standards for Educational and Psychological Tests and Manuals (American Psychological Association, Inc., 1966), p. 13.

[3] Gronlund, loc. cit.

The last of the four types of validity is content validity. The content of a course or curriculum may be broadly defined to include both subject-matter and instructional objectives. The former is concerned with the topics, or subject-matter area, to be covered, and the latter with the behavioral changes sought in pupils. Both of these aspects of content are of concern in determining content validity. More formally, content validity may be defined as the extent to which a test measures a representative sample of the subject-matter content and the behavioral changes under consideration.[1]

For a standardized achievement test to have a high degree of content validity for a subject-matter area, it must be constructed in terms of the same educational objectives as subscribed to by the teacher of that subject matter and must reflect the relative weight of each educational objective as the teacher judges them. The author of an achievement test can hope to orient his instrument toward those educational objectives that appear to be most prevalent.[2]

The idea of judging content validity by comparing the test to a set of goals or objectives is a point of

---

[1] Ibid., p. 62.

[2] Ahmann and Glock, op. cit., p. 300.

agreement among writers in the field of measurement. Ebel wrote:

> Only when the content of education is conceived as a set of goals to be attained, rather than a set of lessons to be studied or as a set of class activities to be carried out, is it educationally useful to seek content validity in a test.[1]

Another writer, Engelhart, wrote:

> In the case of achievement tests the type of validity of greatest concern is content validity, where content refers not merely to the subject-matter covered by the test but to the range of behaviors evaluated.[2]

It has been pointed out above that the validity of an achievement test is of the utmost importance. Furthermore, of the four types, content validity is of greatest importance in achievement tests. The question then to be answered is that of how to carry out this judgment of content validity. Once again Ahmann and Glock stated:

> For a teacher to play the role of pupil and write the answers under conditions similar to those experienced by the pupils is highly recommended. Then an additional item-by-item examination should occur. For each item the question must be asked: In view of my educational objectives, and my method of teaching, is this test item appropriate for my pupils. This is not to be a casual operation. On the contrary, it should be tied directly to the teacher's educational objectives. Although this process is lengthy, it is a necessary one if the content validity of a standardized achievement test is to be assessed.[3]

---

[1]Robert L. Ebel, Educational and Psychological Measurement (Blaisdell Publishing Company, 1967), p. 85.

[2]Max D. Engelhart, "What to Look for in a Review of an Achievement Test," The Personnel and Guidance Journal, XLII (February, 1964), 617.

[3]Ahmann and Glock, op. cit., p. 301.

As has been pointed out in the discussion of content validity, a test has content validity if it tests for the objectives and goals of the area to be tested. It has also been pointed out that these objectives and goals must be stated in terms of behavior changes sought in pupils. This idea is again presented by Mueller.

> We the missionaries and distributors of a product (mathematics) that knows no peer in precision and succinctness, do indeed state our objectives-our goals-in the vaguest sort of rhetoric, couched in soaring platitudes, rich in fervor and zest, but utterly devoid of any measurable criteria.
>
> Inspiring as this sort of thing has been to us in the past--particularly to those of us who already have the religion--the days of nonbehavioral objectives are probably numbered.[1]

Then what is needed to evaluate the Iowa Tests of Educational Development, which is a test of mathematics for pupils in grades nine through twelve, is a set of objectives for secondary mathematics that have been stated in terms of pupil behavior. Such a set of behavioral statements has been developed. In a thesis written by E. M. Oltrogge, he developed a list of behavioral objectives for testing the relative merits of modern and traditional mathematics taught in high school.[2]

---

[1] Francis J. Mueller, "The Revolution at Sputnik-Plus-Ten," The Mathematics Teacher, LX (November, 1967), 705.

[2] Eugene M. Oltrogge, "Behavioral Objectives for Testing the Relative Merits of Modern and Traditional Mathematics in High School" (Masters thesis at Drake University, June, 1965).

These objectives were gathered from statements made by various leaders in the field of mathematics and by a thorough search of the literature. These fourteen objectives are:

The student should

have a knowledge and understanding of mathematical processes, facts and concepts.
have skill in computing with understanding, accuracy, and efficiency.
have the ability to use a general problem solving technique.
understand the logical structure of mathematics and the nature of proof.
use mathematical concepts and processes to discover new generalizations and applications.
recognize and appreciate the role of mathematics in society.
develop study habits essential for independent progress in mathematics.
develop reading skill and vocabulary essential for progress in mathematics.
demonstrate such mental traits as creativity, imagination, curiosity, and visualization.
develop attitudes that lead to appreciation, confidence, respect, initiative, and independence.
be prepared for further work in mathematics or in disciplines which require extensive use of mathematics.
experience a sense of enjoyment, interest, and fascination so as to desire further study in mathematics.
develop the ability to think mathematically through an understanding of "good" mathematics.
develop critical, logical thought, with respect for correct reasoning.[1]

After selecting these fourteen objectives, Oltrogge then classified these objectives under four general behavior divisions. These four divisions are: (1) Knowledge; (2)

---

[1] Ibid., p. 19.

Skills; (3) Attitudes, Interests, and Appreciations; (4) Critical Thinking. In classifying the fourteen objectives into the four behavior divisions, four of the objectives were broken into two parts, with each part being placed under separate behavior divisions. The four objectives are:

1. The student should have a knowledge and understanding of processes, facts, and concepts.
2. The student should recognize and appreciate the role of mathematics in society.
3. The student should develop reading skill and vocabulary essential for progress in mathematics.
4. The student should develop critical, logical thought with respect for correct reasoning.

The following is the rearranged list of the goals under the four categories.

I. Knowledge

   A. The student should have a knowledge of mathematical processes, facts, and concepts.

   B. The student should recognize the role of mathematics in society.

   C. The student should develop vocabulary essential for progress in mathematics.

II. Skills

   A. The student should have skill in computing with understanding, accuracy, and efficiency.

   B. The student should develop reading skill essential for progress in mathematics.

III. Attitudes, Interests, and Appreciations

A. The student should appreciate the role of mathematics in society.

B. The student should demonstrate such mental traits as creativity, imagination, curiosity, and visualization.

C. The student should develop attitudes that lead to appreciation, confidence, respect, initiative, and independence.

D. The student should experience a sense of enjoyment, interest, and fascination so as to desire further study in mathematics.

E. The student should have respect for correct reasoning.

IV. Critical Thinking

A. The student should have an understanding of mathematical processes, facts, and concepts.

B. The student should have the ability to use a general problem solving technique.

C. The student should understand the logical structure of mathematics and the nature of proof.

D. The student should use concepts and processes to discover new generalizations and applications.

E. The student should develop study habits essential for independent progress in mathematics.

F. The student should be prepared for further work in mathematics, or in disciplines which require extensive use of mathematics.

G. The student should develop the ability to think mathematically through an understanding of "good" mathematics.

H. The student should develop critical, logical thought.

The next step used by Oltrogge was to write the compound statements in the form of several simple statements. An example of this would be changing the first statement:

I. Knowledge

A. The student should have a knowledge of mathematical processes, facts, and concepts.

to three statements. These three statements would look like this.

I. Knowledge

A-1. The student knows mathematical processes.

A-2. The student knows mathematical facts.

A-3. The student knows mathematical concepts.

This process was carried out for each statement that was in the form of a compound sentence. After this process had been completed, the final step was to follow each of the simple goal statements with behavior statements. The code used by Oltrogge for this final step was: (1) The

Roman numeral indicates the major behavior divisions; (2) The capital letter indicates the objectives from which it was derived; and (3) The Arabic numeral indicates the simple behavioral statement.

Due to the complexity of the area of Attitudes, Interests, and Appreciations, and the fact that it relies so heavily upon the area of psychology, it was omitted when the list of simple behavior statements were given.

The following is the final break down of the fourteen objectives into typical student behaviors.

I. Knowledge

A-1. The student knows mathematical processes.

The student:

(1) Applies processes to problems and new situations which require their use

(2) States essential differences between processes

(3) States processes

(4) Uses parts of the processes

(5) Uses processes to check and verify answers which have made use of processes

(6) Classifies processes

(7) Applies processes in making predictions

(8) Uses processes in solving problems of a related field, whenever this is possible

(9) Reviews situations and tells those in which a certain process or processes can be used

(10) Corrects wrongly stated processes

(11) Identifies wrong applications and corrects them

(12) Tells which of a group of characteristics belong to a certain process

A-2. The student knows mathematical facts.

The student:

(1) Makes use of the facts in problems and new situations which require their use

(2) States the facts

(3) Contrasts differences between facts

(4) Identifies situations in which facts have been used

(5) Uses facts to check and verify solutions that have made use of those same facts

(6) Applies facts in making predictions

(7) Uses facts, where applicable, to solve problems in a related field

(8) Reviews a group of situations and indicates those in which a certain fact or facts can or cannot be used

(9) Corrects wrongly stated facts

(10) Identifies wrong references to facts and

corrects them

(11) Classifies facts

(12) Uses facts to solve problems in everyday life

A-3. The student knows mathematical concepts.

The student:

(1) Applies concepts to problems and new situations which require their use

(2) States the characteristics of the concepts

(3) States essential differences between the concepts

(4) Uses the necessary concepts to check or verify probable solutions

(5) Classifies concepts into categories

(6) Uses concepts in making predictions or estimates

(7) Uses concepts in problems of a related field, whenever this is possible

(8) Tells if additional data is required in order to meet the needed characteristics of a given concept

(9) States evidence which indicates that a given concept should be used

(10) Corrects wrongly stated concepts

(11) Indicates true and false items which are

based on a concept

(12) Tells which of a group of characteristics listed belong to a certain concept or group of concepts

B-1. The student knows the role of mathematics in society.

The student:

(1) Tells the value of applying the logical thought of mathematics to non-mathematical situations

(2) Tells the importance of identifying assumptions and conclusions in situations in society

(3) Tells the importance of examining the validity of reasoning processes in situations arising in society

(4) States the association between mathematics and the practical arts

(5) Describes the role of mathematics in technology

(6) Describes the role of mathematics in industry

(7) Describes the relationship between mathematics and science

(8) Describes why mathematics is indispensable for further progress

(9) Discusses the importance of mathematics in the broad categories of professions, trades, and technical fields.

(10) Defends the importance of mathematics in history and in current affairs

(11) Describes the mathematization of knowledge which has been increasingly evident, not only in the fields of physics and chemistry, but in fields which until recently weren't mathematical at all

(12) Explains how mathematics helps us understand our environment

(13) Describes the importance of mathematical thought in philosophy and as a factor in the evolution of civilization

(14) Interprets the word "elegance" as it applies to the beauty of mathematics

(15) Lists practical values of mathematics

C-1. The student knows the vocabulary essential for progress in mathematics.

The student:

(1) Uses mathematics vocabulary words in problems which require their use

(2) Describes the relationship between mathematics vocabulary words

(3) Traces vocabulary items to related items

(4) Answers simple questions about given vocabulary items

(5) Observes two or more situations which have certain common aspects and names the common aspects with the proper vocabulary item

(6) Defines vocabulary items

(7) Tells if additional information or data is needed to satisfy the definition of a vocabulary item

(8) Corrects wrongly stated vocabulary definitions

(9) Matches general vocabulary items with more specific ones

(10) States situations which make use of given vocabulary items

(11) Tells the conditions needed for the use of a vocabulary item

(12) Indicates which of a given group of situations are describable by a vocabulary item

II. Skills

A-1. The student is skillful in computing with understanding.

The student:

(1) Computes with speed

(2) Indicates knowledge of arithmetic insights

(3) Explains the "why" of algorithms

(4) States relations between numbers

(5) Estimates solutions

(6) Uses algorithms

(7) Uses numerical processes

(8) Indicates the plausibility of solutions

(9) Uses new symbols when necessary

(10) Selects the correct algorithms

(11) Indicates when an answer has been obtained

(12) Interprets solutions

(13) Analyzes problem situations and uses the appropriate processes

(14) Uses vocabulary related to computing

(15) Corrects wrong computations

(16) States the principles involved when using a certain type of computation

A-2. The student is accurate with computations.

The student:

(1) Indicates errors in inaccurate solutions

(2) Solves problems correctly

(3) Performs motor skills satisfactorily

(4) Corrects wrong computations

(5) Validates solutions

(6) Indicates plausibility of solutions

(7) Computes accurately

(8) Selects the correct algorithms

(9) Indicates when an answer has been obtained

(10) Analyzes problem situations and uses the appropriate processes

A-3. The student is efficient with computations.

The student:

(1) Uses algorithms

(2) Uses numerical processes

(3) Computes efficiently

(4) Uses new symbols when necessary

(5) Selects the correct algorithms

(6) Indicates when an answer has been obtained

(7) Analyzes problem situations and uses the appropriate processes

(8) Uses processes which are most effective and least wasteful of time

(9) Completes large numbers of problems in a limited amount of time

B-1. The student develops reading skill essential for progress in mathematics.

The student:

(1) Uses an adequate vocabulary

(2) Understandably reads problems involving insufficient data

(3) Understandably reads problems involving superflous data

(4) Understandably reads problems involving induction or deduction

(5) Understandably reads problems involving extrapolation

(6) Understandably reads problems involving insight

(7) Understandably reads problems involving transfer

(8) Understandably reads problems involving familiar or unfamiliar situations

(9) Analyzes proofs and indicates wrong statements

(10) Makes specific deductions

(11) Makes accurate and well founded conjectures

(12) Makes well founded generalizations

(13) Transforms problems into an accessible formulation of the problem

(14) Applies different word meanings for different problems

(15) Solves the exact problem stated

(16) Demonstrates mathematical reading accuracy

(17) Reads with comprehension

III. Attitudes, Interests, and Appreciations

A. As previously indicated, the writer will not attempt to break the goals into behaviors in the area of Attitudes, Interests, and Appreciations.

IV. Critical Thinking

A-1. The student understands the mathematics processes. The student:

(1) Applies processes to problems which require their use

(2) States the processes

(3) Contrasts processes

(4) Makes statements about the correctness of given solutions with respect to the processes used

(5) Critically evaluates arguments which make use of processes

(6) Uses processes to check or verify answers

(7) Classifies processes into categories

(8) Indicates common processes used in solutions to two or more problems

(9) Traces processes to more elementary processes

(10) Uses pertinent vocabulary

(11) States if additional data is needed in order to use a given process

(12) Corrects wrongly stated processes

(13) Classifies items true or false with respect

to the use of given processes

(14) Shows why a certain process can or should be used

(15) Tells which of a group of characteristics belong to a certain process

(16) Classifies general processes with more specific ones

A-2. The student understands mathematical facts.

The student:

(1) Applies facts to problems and new situation require their use

(2) States the facts

(3) States the differences between facts

(4) Makes statements about the correctness of given solutions which make use of a given set of facts

(5) Criticizes arguments which make use of facts

(6) Uses facts to verify or check procedures

(7) Indicates basic relationships between facts

(8) Tells the common facts used in two or more situations

(9) Uses facts to make predictions

(10) Uses facts to solve problems in a related field, whenever this is possible

(11) Uses vocabulary pertinent to the facts

(12) Corrects wrongly stated facts

(13) Determines items based on facts as true or false

(14) Identifies wrong references to facts and corrects them

A-3. The student understands mathematical concepts. The student:

(1) Applies concepts to problems and new situations which require their use

(2) States the characteristics of a given concept

(3) States essential differences between concepts

(4) Makes statements about the correctness of given solutions with respect to the concepts which have been used

(5) Criticizes arguments which make use of concepts

(6) Uses necessary concepts to check or verify probable solutions

(7) Indicates common concepts used in two or more situations

(8) Uses concepts to help in making predictions or estimates

(9) Traces concepts to more elementary, related concepts

(10) Uses concepts in problems of a related field,

whenever this is possible.

(11) Uses the pertinent vocabulary of concepts

(12) Indicates whether a certain concept can or should be used

(13) Corrects wrongly stated concepts

(14) Classifies items based on applications of concepts as true or false

(15) Classifies specific concepts with more general ones

B-1. The student demonstrates general problem solving techniques.

The student:

(1) Solves the exact problem stated

(2) Applies different word meanings for different problems

(3) Selects appropriate problem solving techniques for a given problem situation

(4) Indicates relevant and irrelevant information

(5) States what added information, if any, is needed

(6) Transforms problems into an accessible formulation of the problem

(7) Selects suitable notation for a given technique

(8) Selects suitable symbolism for a given technique

(9) Uses the correct algorithms

(10) Indicates when new ideas are needed

(11) Indicates the need to look for a counter example

(12) Indicates knowledge of when an answer has been obtained

(13) Validates solutions

(14) States possible significance of solutions

(15) States possible generalizations of solutions

(16) Indicates recognition of patterns

(17) Changes approach when necessary

(18) States essential differences between problems which require different techniques

C-1. The student understands the logical structure of mathematics.

The student:

(1) Answers simple questions about structure

(2) States the algebraic aspects of structure

(3) Applies the algebraic aspects of structure to problems

(4) Criticizes arguments which make use of structure aspects

(5) Uses aspects of structure to check or verify

solutions

(6) States the order aspects of structure

(7) States examples of paired elements for which a given relation is true or false

(8) States the defined relations between objects of the set being considered

(9) Determines truth or falsity of given statements with respect to the original definitions and postulates

(10) Indicates statements which can be derived from given information

(11) Shows proficiency in areas of numerical approximation

(12) Uses inequalities

(13) Uses sequential development of axioms

(14) States the material used to develop algorithms

(15) Indicates recognition of patterns

(16) Verbalizes arithmetic structure

(17) Validates identities

(18) Uses vocabulary pertinent to structure

(19) Indicates if additional data is needed in order to draw a conclusion from a given set of definitions and assumptions

(20) Corrects wrongly stated aspects of structure

(21) Indicates false implications which result from improper interpretation of certain aspects of structure

C-2. The student understands the nature of proof. The student:

(1) Classifies proofs into indirect and direct categories

(2) Tells relationship between statements used in a proof

(3) Classifies general statements with specific ones

(4) Indicates recognition of necessary and sufficient conditions

(5) Points out irrelevant material in a proof presented to him

(6) Places statements and reasons under the correct heading if given a "proof" in which they are scrambled

(7) Indicates the type of reasoning used in a given question-answer situation

(8) Tells objections to types of reasoning which cannot be used in a given problem situation

(9) Indicates flaws in a given "proof"

(10) Indicates the point at which a proof is complete

(11) Tells if statements are true, false, or either true or false on the basis of the given information

(12) Refers to a given proof and correctly answers questions about changes in the order of steps which could be made without affecting the validity of the proof

(13) Refers to a given proof and tells if different reasons could be used for a chosen statement

(14) Refers to a given proof and determines if the proof remains valid if given steps in the proof are interchanged

(15) Correctly answers questions which are implications of a theorem which is presented to him

(16) Tells if a proof is still valid after changing a certain condition

(17) Considers a statement which is related to a given theorem and tells if it is true or false on the basis of the theorem

(18) Indicates the chain of reasoning in a proof--that is, he tells which statements in a proof are needed to imply a later statement

(19) Provided a given "theorem" is untrue,

states the first step of a possible proof which is wrong

D-1. The student uses mathematical processes and concepts to discover new generalizations.

The student:

(1) Indicates recognition of patterns

(2) Makes conjectures

(3) Speculates about unknown areas

(4) Is resourceful in problem solving

(5) Classifies indicated statements as always true, sometimes true, or never true

(6) Finds alternative methods

(7) Attempts to be his own teacher

(8) Experiments with possibilities

(9) Organizes details which are related to a given item

(10) Eliminates items unrelated to a given topic

(11) Makes specific deductions from general statements

(12) Uses ideas of necessary and sufficient conditions

(13) Explains logical connections between related items

(14) Relates information to the larger whole

(15) Makes proper use of intuition

(16) Considers statements and indicates whether or not they follow from a certain theorem

(17) Indicates statements which can be derived from given information

(18) Indicates if additional data is needed in order to draw a conclusion from a given set of definitions and assumptions

(19) States differences between concepts

(20) Indicates common concepts and processes used in two or more situations

(21) States differences between processes

(22) Traces concepts to more elementary ones

D-2. The student uses mathematical concepts and processes to discover new applications.

The student:

(1) States differences between closely related applications

(2) Indicates common concepts and processes used in two or more situations

(3) States differences between processes

(4) States differences between concepts

(5) Discovers new applications

(6) Considers problems and tells if they could be solved by applying certain indicated processes and/or concepts

(7) Looks for alternative methods of doing problems

(8) Experiments with mathematical possibilities

(9) Makes mathematical conjectures

(10) Speculates about unknown problems

(11) Uses mathematical intuition

E-1. The student develops study habits essential for independent progress in mathematics.

The student:

(1) Indicates plausibility of solutions

(2) Uses new symbols when necessary

(3) Selects the correct algorithms

(4) Indicates when an answer has been obtained

(5) Uses the appropriate mathematical material after analyzing problem situations

(6) Makes specific deductions from general statements

(7) Makes generalizations

(8) Critically evaluates proof

(9) Indicates recognition of patterns and relationships

(10) Changes approach when necessary

(11) Exhibits perseverance

(12) Traces vocabulary items to related ones

(13) Organizes data

(14) Looks for counterexamples when this is called for

(15) Relates information to the larger whole

(16) Plausibly estimates

(17) Applies mathematical judgment

(18) Computes accurately

(19) Verifies solutions

(20) Validates identities

F-1. The student is able to continue further work in mathematics.

The student:

(1) Exhibits work consistent with the next course in mathematics

(2) States the type of mathematics which follows his present course

(3) Performs well in future mathematics courses

(4) Consistently demonstrates the ability to use the mathematical knowledge he possesses to discover results which are new to him

(5) Illustrates the type of behavior which is exemplified under the simple goal statements in this report

(6) Uses a good mathematics vocabulary

(7) Selects appropriate mathematics for problems with which he is confronted

F-2. The student is able to continue further work in disciplines which require extensive use of mathematics.

The student:

(1) Capably uses the mathematics associated with future physics courses

(2) Capably uses the mathematics associated with future chemistry courses

(3) Capably uses the mathematics associated with future biology courses

(4) Capably uses mathematics associated with future courses in the social sciences

(5) Capably uses mathematics associated with future astronomy courses

(6) Indicates an understanding of the qualities of good data

(7) Uses methods best adapted for the problems at hand

(8) Selects appropriate mathematics for problems with which he is confronted

(9) Performs well in future science courses.

G-1. The student thinks mathematically.

The student:

(1) Thinks and argues about mathematics for himself

(2) Indicates chains of reasoning used for deriving a statement

(3) Matches general statements with related specific ones

(4) Explains logical connections between related items

(5) Uses correct ideas of implications

(6) Applies correct notions of postulates

(7) Explains mathematics as a system with the various parts taking their places as contributing parts of the whole system

(8) Uses ideas of necessary and sufficient conditions

(9) Indicates untrue implications

(10) Makes specific deductions from given general statements

(11) Makes well founded generalizations

(12) Solves problems which involve deduction

(13) Relates information to the larger whole

(14) Properly uses intuition

(15) Makes accurate and well founded conjectures

(16) Refers to a given proof and determines if the proof remains valid if certain steps in the proof are interchanged

(17) Correctly answers questions which are

implications of a theorem presented to him

(18) Indicates if additional data is needed in order to draw a conclusion from a given set of definitions and assumptions

(19) Tells situations in society where assumptions and conclusions can be identified

G-2. The student understands "good" mathematics.

The student:

(1) Relates information to the larger whole

(2) Explains mathematics as a system, with the various parts taking their places as contributing parts of the whole system

(3) Uses algebraic aspects of structure

(4) Applies order aspects of mathematics

(5) Uses elementary aspects of number theory

(6) Answers the "why" of algorithms

(7) Explains logical connections between related items

(8) Validates identities

(9) Constructs simple proofs

(10) States relations between properties

(11) Uses correct ideas of implications

(12) Applies correct notions of postulates

(13) Uses the idea of equivalence

(14) Uses the idea of undefined terms

(15) Tells the relationship between a statement and its converse

(16) Uses ideas of necessary and sufficient conditions

(17) Solves problems which involve transfer

(18) Solves problems which involve insight

(19) Solves problems which involve deduction

(20) Makes generalizations

(21) Critically evaluates proof

(22) Indicates recognition of patterns

(23) Makes specific deductions from general statements

(24) Tells untrue implications

H-1. The student is able to critically examine mathematical material.

The student:

(1) Thinks and argues about mathematics for himself

(2) Describes mathematics as a series of interrelated parts which are dominated by the character of the whole

(3) Uses ideas of necessary and sufficient conditions

(4) Refers to a given proof and determines if the proof remains valid if certain steps in

the proof are interchanged

(5) Correctly answers questions which are implications of a theorem presented to him

(6) Critically evaluates proof

(7) Indicates recognition of patterns and relationships

(8) Looks for counterexamples when this is called for

(9) Validates identities

(10) Verifies solutions

(11) Constructs simple proofs

(12) States relations between properties

(13) Classifies proofs into direct and indirect categories

(14) Corrects wrongly stated concepts, processes, or facts

(15) States evidence which indicates that a given concept, process, or fact should be used

H-2. The student thinks logically.

The student:

(1) Explains logical connections between related items

(2) Matches general statements with related specific ones

(3) Uses correct ideas of implication

(4) Uses ideas of necessary and sufficient conditions

(5) Indicates untrue implications

(6) Makes specific deductions from a given general statement

(7) Makes well founded generalizations

(8) Solves problems which involve deduction

(9) Relates information to the larger whole

(10) Makes accurate and well founded conjectures

(11) Correctly answers questions which are implications of a theorem presented to him

(12) States the relationship between a statement and its converse

(13) Indicates the chain of reasoning in a proof--that is, he tells which statements in a proof are needed to imply a later statement

(14) Identifies assumptions and conclusions underlying a statement

(15) Tells the value of applying the logical thought of mathematics to non-mathematical situations[1]

---

[1] Eugene M. Oltrogge, "Behavioral Objectives for Testing the Relative Merits of Modern and Traditional Mathematics in High School" (Masters thesis at Drake University, Des Moines, Iowa, June, 1965), pp. 38-65.

This completes the listing of simple behaviors that should be the goals of a secondary mathematics program. The manner in which these behavior statements are used is explained in the next section.

II. PROCEDURE

After selecting the Iowa Tests of Educational Development as the test to be evaluated, a copy of Form X-4 was obtained from the State University of Iowa. The author then chose a problem from the test to be evaluated. This problem was then typed on a separate sheet of paper and solved on that paper with all operations shown. If more than one method of solution could be used, the other methods were also shown on the same page.

There remained, however, the need for a means of checking the behaviors required in the test against those suggested by Oltrogge. This was accomplished by devising a checklist numbered in the same way as the behaviors listed on pages sixteen through forty-two above. A copy of the checklist is on the following page. The writer then proceded to examine each problem solution sheet against the entire list of 360 behaviors. When a behavior was required in the solution, a check mark was placed by the number of the behavior on the checklist.

The same procedure was followed for each of the

TABLE I

CHECKLIST FOR RECORDING BEHAVIORS TESTED

I.
A-1
1
2
3
4
5
6
7
8
9
10
11
12

A-2
1
2
3
4
5
6
7
8
9
10
11
12

A-3
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

B-1
1
2
3
4
5
6

7
8
9
10
11
12
13
14
15

C-1
1
2
3
4
5
6
7
8
9
10
11
12

II.
A-1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

A-2
1
2
3
4
5
6
7
8
9
10
11
12
13

IV.
A-1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

B-1
1
2
3
4
5
6
7
8
9
10
11

12
13
14
15

C-2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

D-1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19

D-2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22

E-1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

F-1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

F-2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

G-1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

G-2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

H-1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24

H-2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

fifty-three questions. This provided the writer with a record of all of the behavioral operations required for the solution of the problems in the test.

When all fifty-three problem solutions had been compared with a complete list of behaviors and the results recorded on a separate checklist, the analysis of the results was made. This was done by counting the number of problems that required the first behavior. This number was recorded on a final checklist beside the number of that behavior statement. This was continued for each behavior until all 360 behaviors had totals. Some behaviors were required in all fifty-three problem solutions and other behaviors being required in fewer problem solutions or none.

It was after these totals were counted that it was possible to group the behavior statements. The four groups used were: (1) Those behaviors that were required by forty or more problem solutions; (2) Those behaviors that were required by twenty-five through thirty-nine problem solutions; (3) Those behaviors that were required by one through fifteen problem solutions; and (4) Those behaviors that were not required by any problem solution. In analyzing the totals it was found that there were no objectives that were required by sixteen through twenty-four problem solutions.

After the behaviors were grouped, an analysis was made as to the type of behaviors found in each group and as to why they were found in that group.

CHAPTER II

GROUPING OF BEHAVIORS

In this chapter the behaviors will be grouped according to the number of problem solutions that required their use. Before the behaviors can be grouped, a knowledge of how many problem solutions required the use of each of the 360 behavioral statements must be known. This information was obtained by going through each of the fifty-three checklists and counting the number of checks that each behavioral statement had. This information is in Table II on the following page. In a further attempt to group the behaviors, a tabulation was made as to the number of behaviors that were required for all fifty-three problem solutions, the number of behaviors required for fifty-two problem solutions, and so on for all numbers zero through fifty-three. This information is in Table III. In looking at Table III, it appears that the behaviors tend to fall into four groups. First, there are those that were required by forty or more problem solutions. Second, there are those behaviors that were required by twenty-five through thirty-nine problem solutions. Third, there are those behaviors that were required by one through fifteen problem solutions. Finally, there were those behaviors that were not required by any of the problem solutions. There were no behaviors that were

TABLE II

FINAL CHECKLIST OF BEHAVIORS TESTED

NOTE: This checklist has been expanded to two pages for ease of reading. The first numbers are the same as the checklist of page forty-four and the numbers beside these numbers are the number of problem solutions that required that behavior.

I. A-1

1-53
2-0
3-0
4-44
5-32
6-0
7-2
8-29
9-1
10-0
11-0
12-0

A-2

1-51
2-0
3-0
4-0
5-34
6-3
7-34
8-0
9-0
10-0
11-0
12-44

A-3

1-51
2-0
3-0
4-33
5-0
6-5
7-34
8-0
9-2
10-0
11-1
12-0

B-1

1-0
2-0
3-1
4-0
5-0
6-0
7-0
8-0
9-0
10-0
11-0

C-1

1-26
2-0
3-0
4-1
5-0
6-0
7-0
8-0
9-0
10-0
11-0
12-0

II. A-1

1-42
2-3
3-0
4-15
5-7
6-45
7-47
8-46
9-3
10-46
11-53
12-37
13-52
14-0
15-4
16-0

A-2

1-0
2-48
3-3
4-3
5-32
6-44
7-42
8-46
9-52
10-51

A-3

1-46
2-48
3-43
4-5
5-46
6-53
7-52
8-36
9-48

B-1

1-36
2-0
3-9
4-35
5-1
6-1
7-28
8-48
9-0
10-37
11-1
12-0
13-41
14-1
15-39
16-30
17-32

IV. A-1

1-53
2-0
3-1
4-0
5-0
6-32
7-0
8-0
9-0
10-0
11-0
12-0
13-0
14-0
15-0
16-0

A-2

1-48
2-1
3-0
4-7
5-0
6-30
7-1
8-0
9-4
10-33
11-1
12-0
13-0
14-0

A-3

1-51
2-0
3-0
4-0
5-0
6-40
7-0
8-8
9-0
10-31
11-1
12-1
13-0
14-0
15-0

B-1

1-50
2-0
3-50
4-0
5-0
6-43
7-7
8-9
9-47
10-1
11-0
12-50
13-31
14-0
15-0
16-1
17-0
18-0

C-1

1-4
2-0
3-11
4-0
5-9
6-0
7-0
8-0
9-0
10-7
11-1
12-1
13-0
14-0
15-2
16-1

TABLE II (Continued)

IV.

C-1	16-0	F-1	G-2	H-2
	17-5			
17-0	18-1	1-0	1-0	1-0
18-1	19-0	2-0	2-0	2-5
19-1	20-0	3-0	3-12	3-0
20-0	21-0	4-0	4-1	4-0
21-0	22-0	5-0	5-29	5-0
		6-11	6-0	6-31
C-2	D-2	7-50	7-0	7-0
			8-0	8-37
1-0	1-0	F-2	9-0	9-0
2-0	2-0		10-0	10-2
3-0	3-0	1-5	11-0	11-0
4-0	4-1	2-2	12-0	12-0
5-0	5-0	3-2	13-1	13-0
6-0	6-0	4-7	14-0	14-0
7-0	7-2	5-1	15-0	15-1
8-0	8-1	6-0	16-0	
9-0	9-2	7-51	17-25	
10-0	10-0	8-53	18-0	
11-0	11-0	9-0	19-34	
12-0			20-0	
13-0	E-1	G-1	21-0	
14-0			22-1	
15-0	1-46	1-0	23-32	
16-0	2-2	2-0	24-0	
17-0	3-46	3-7		
18-0	4-53	4-0	H-1	
19-0	5-44	5-1		
	6-30	6-0	1-0	
D-1	7-0	7-0	2-0	
	8-0	8-0	3-0	
1-4	9-2	9-0	4-0	
2-3	10-1	10-30	5-0	
3-0	11-0	11-0	6-0	
4-49	12-0	12-36	7-1	
5-0	13-5	13-0	8-0	
6-0	14-0	14-0	9-0	
7-0	15-0	15-1	10-32	
8-0	16-3	16-0	11-0	
9-1	17-45	17-0	12-0	
10-3	18-42	18-0	13-0	
11-26	19-37	19-0	14-0	
12-0	20-0		15-0	
13-0				
14-0				
15-0				

TABLE III

QUANTITY OF BEHAVIORAL STATEMENTS THAT WERE REQUIRED BY AN EXACT QUANTITY OF PROBLEM SOLUTIONS

Quantity of Behaviors	Quantity of Problem Solutions	Quantity of Behaviors	Quantity of Problem Solutions
6 were required for all	53	2	26
3 were required in	52	1	25
5	51	0	24
4	50	0	23
1	49	0	22
5	48	0	21
2	47	0	20
7	46	0	19
2	45	0	18
4	44	0	17
2	43	0	16
3	42	1	15
1	41	0	14
1	40	0	13
1	39	1	12
0	38	2	11
4	37	0	10
3	36	3	9
1	35	1	8
4	34	6	7
2	33	0	6
6	32	6	5
3	31	4	4
4	30	8	3
2	29	10	2
1	28	35	1
0	27	203	0

required by sixteen through twenty-four problem solutions.

As pointed out earlier, the behavioral statements tend to fall into four groups. The first group included those behaviors that were required by forty or more problem solutions. Those behaviors that are in the first group will now be listed. The number of problem solutions requiring the behavior listed is shown by the number in parenthesis that follows the behavior. This number could also be found by looking at Table II. The behaviors are numbered in the same manner that they were in the list given on pages sixteen through forty-two.

I. Knowledge

A-1 The student knows mathematical processes.

The student:

(1) Applies processes to problems and new situations which require their use (53)

(4) Uses parts of the processes (44)

A-2 The student knows mathematical facts.

The student:

(1) Makes use of the facts in problems and new situations which require their use (51)

(12) Uses facts to solve problems in everyday life (44)

A-3 The student knows mathematical concepts.

The student:

(1) Applies concepts to problems and new situations which require their use (51)

II. Skills

A-1 The student is skillful in computing with understanding.

The student:

(1) Computes with speed (42)

(6) Uses algorithms (45)

(7) Uses numerical processes (47)

(8) Indicates the plausibility of solutions (46)

(10) Selects the correct algorithms (46)

(11) Indicates when an answer has been obtained (53)

(13) Analyzes problem situations and uses the appropriate processes (52)

A-2 The student is accurate with computations.

The student:

(2) Solves problems correctly (48)

(6) Indicates plausibility of solutions (44)

(7) Computes accurately (42)

(8) Selects the correct algorithms (46)

(9) Indicates when an answer has been obtained (52)

(10) Analyzes problem situations and uses the appropriate processes (51)

A-3 The student is efficient with computations.

The student:

(1) Uses algorithms (46)

(2) Uses numerical processes (48)

(3) Computes efficiently (43)

(5) Selects the correct algorithms (46)

(6) Indicates when an answer has been obtained (53)

(7) Analyzes problem situations and uses the appropriate processes (52)

(9) Completes large numbers of problems in a limited amount of time (48)

B-1 The student develops reading skill essential for progress in mathematics.

The student:

(8) Understandably reads problems involving transfer (48)

(13) Transforms problems into an accessible formulation of the problem (41)

IV. Critical Thinking

A-1 The student understands the mathematical processes.

The student:

(1) Applies processes to problems which require their use (53)

A-2 The student understands mathematical facts.

The student:

(1) Applies facts to problems and new situations which require their use (48)

A-3 The student understands mathematical concepts.

The student:

(1) Applies concepts to problems and new situations which require their use (51)

(6) Uses necessary concepts to check or verify probable solutions (40)

B-1 The student demonstrates general problem solving techniques.

The student:

(1) Solves the exact problem stated (50)

(3) Selects appropriate problem solving techniques for a given problem situation (50)

(6) Transforms problems into an accessible formulation of the problem (43)

(9) Uses the correct algorithms (47)

(12) Indicates knowledge of when an answer has been obtained (50)

D-1 The student uses mathematical processes and concepts to discover new generalizations.

The student:

(4) Is resourceful in problem solving (49)

E-1 The student develops study habits essential for independent progress in mathematics.

The student:

(1) Indicates plausibility of solutions (46)

(3) Selects the correct algorithms (46)

(4) Indicates when an answer has been obtained (53)

(5) Uses the appropriate mathematical material after analyzing problem situations (44)

(17) Applies mathematical judgment (45)

(18) Computes accurately (42)

F-1 The student is able to continue further work in mathematics.

The student:

(7) Selects appropriate mathematics for problems with which he is confronted (50)

F-2 The student is able to continue further in disciplines which require extensive use of mathematics.

The student:

(7) Uses methods best adapted for the problems at hand (51)

(8) Selects appropriate mathematics for problems with which he is confronted (53)

This completes the first group of behaviors. In the second group are those behaviors that were required in twenty-five through thirty-nine problem solutions. They are listed in the same manner as those in the preceding section.

I. Knowledge

A-1 The student knows mathematical processes.

The student:

(5) Uses processes to check and verify answers which have made use of processes (32)

(8) Uses processes in solving problems of a related field, whenever this is possible (29)

A-2 The student knows mathematical facts.

The student:

(5) Uses facts to check and verify solutions that have made use of those same facts (34)

(7) Uses facts, where applicable, to solve problems in a related field (34)

A-3 The student knows mathematical concepts.

The student:

(4) Uses the necessary concepts to check or verify probable solutions (33)

(7) Uses concepts in problems of a related field, whenever this is possible (34)

C-1 The student knows the vocabulary words in problems which require their use.

The student:

(1) Uses mathematics vocabulary words in problems which require their use (26)

II. Skills

A-1 The student is skillful in computing with understanding.

The student:

(12) Interprets solutions (37)

A-2 The student is accurate with computations.

The student:

(5) Validates solutions (32)

A-3 The student is efficient with computations.

The student:

(8) Uses processes which are most effective and least wasteful of time (36)

B-1 The student develops reading skill essential for progress in mathematics.

The student:

(1) Uses an adequate vocabulary (36)

(4) Understandably reads problems involving induction or deduction (35)

(7) Understandably reads problems involving transfer (28)

(10) Makes specific deductions (37)

(15) Solves the exact problem stated (39)

(16) Demonstrates mathematical reading accuracy (30)

(17) Reads with comprehension (32)

IV. Critical Thinking

A-1 The student understands the mathematics processes.

The student:

(6) Uses processes to check or verify answers (32)

A-2 The student understands mathematical facts.

The student:

(6) Uses facts to verify or check procedures (30)

(10) Uses facts to solve problems in a related field, whenever this is possible (33)

A-3 The student understands mathematical concepts.

The student:

(10) Uses concepts in problems of a related field, whenever this is possible (31)

B-1 The student demonstrates general problem solving techniques.

The student:

(13) Validates solutions (31)

D-1 The student uses mathematical processes and concepts to discover new generalizations.

The student:

(11) Makes specific deductions from general statements (26)

E-1 The student develops study habits essential for independent progress in mathematics.

The student:

(6) Makes specific deductions from general statements (30)

(19) Verifies solutions (37)

G-1 The student thinks mathematically.

The student:

(10) Makes specific deductions from given general statements (30)

(12) Solves problems which involve deduction (36)

G-2 The student understands good mathematics.

The student:

(5) Uses elementary aspects of number theory (29)

(17) Solves problems which involve transfer (25)

(19) Solves problems which involve deduction (34)

(23) Makes specific deductions from general statements (32)

H-1 The student is able to critically examine mathematical material.

The student:

(10) Verifies solutions (32)

H-2 The student thinks logically.

The student:

(6) Makes specific deductions from a given general statement (31)

(8) Solves problems which involve deduction (37)

This completes the second group of behavioral statements. In checking the list in Table III it is found that there are no behaviors that were required by sixteen through twenty-four problem solutions. Therefore, the grouping to be made next is that of those behavioral statements that were required by one through fifteen problem solutions. The following statements are in that group.

I. Knowledge

A-1 The student knows mathematical processes.

The student:

(7) Applies processes in making predictions (2)

(9) Reviews situations and tells those in which a certain process can be used (1)

A-2 The student knows mathematical facts.

The student:

(6) Applies facts in making predictions or estimates (5)

A-3 The student knows mathematical concepts.

The student:

(6) Uses concepts in making predictions or estimates (5)

(9) States evidence which indicates that a given concept should be used (2)

(11) Indicates true and false items which are based on a concept (1)

B-1 The student knows the role of mathematics in society.

The student:

(3) Tells the importance of examining the validity of reasoning processes in situations arising in society (1)

(12) Explains how mathematics helps us understand our environment (1)

C-1 The student knows the vocabulary essential for progress in mathematics.

The student:

(4) Answers simple questions about given vocabulary items (1)

II. Skills

A-1 The student is skillful in computing with understanding.

The student:

(2) Indicates knowledge of arithmetic insights (3)

(4) States relations between numbers (15)

(5) Estimates solutions (7)

(9) Uses new symbols when necessary (3)

(15) Corrects wrong computations (4)

A-2 The student is accurate with computations.

The student:

(3) Performs motor skills satisfactorily (3)

(4) Corrects wrong computations (3)

A-3 The student is efficient with computations.

The student:

(4) Uses new symbols when necessary (5)

B-1 The student develops reading skill essential for progress in mathematics.

The student:

(3) Understandably reads problems involving induction or deduction (9)

(5) Understandably reads problems involving extrapolation (1)

(6) Understandably reads problems involving insight (1)

(11) Makes well founded generalizations (1)

(14) Applies different word meaning for different problems (1)

IV. Critical Thinking

A-1 The student understands the mathematics processes.

The student:

(3) Contrasts processes (1)

A-2 The student understands mathematical facts.

The student:

(2) States the facts (1)

(4) Makes statements about the correctness of given solutions which make use of a given

set of facts (7)

(7) Indicates basic relationships between facts (1)

(9) Uses the facts to make predictions (4)

(11) Uses vocabulary pertinent to the facts (1)

A-3 The student understands mathematical concepts.

The student:

(8) Uses concepts to help in making predictions or estimates (8)

(11) Uses the pertinent vocabulary of concepts (1)

(12) Indicates whether a certain concept can or should be used (1)

B-1 The student demonstrates general problem solving techniques.

The student:

(7) Selects suitable notation for a given technique (7)

(8) Selects suitable symbolism for a given technique (9)

(10) Indicates when new ideas are needed (1)

(16) Indicates recognition of patterns (1)

C-1 The student understands the logical structure of mathematics.

The student:

(1) Answers simple questions about structure (4)

(3) Applies the algebraic aspects of structure to problems (11)

(5) Uses aspects of structure to check or verify solutions (9)

(10) Indicates statements which can be derived from given information (7)

(11) Shows proficiency in area of numerical approximation (1)

(12) Uses inequalities (1)

(15) Indicates recognition of patterns (2)

(16) Verbalizes arithmetic structure (1)

(18) Uses vocabulary pertinent to structure (1)

(19) Indicates if additional data is needed in order to draw a conclusion from a given set of definitions and assumptions (1)

D-1 The student uses mathematical processes and concepts to discover new generalizations.

The student:

(1) Indicates recognition of patterns (4)

(2) Makes conjectures (3)

(9) Organizes details which are related to a given item (1)

(10) Eliminates items unrelated to a given topic (3)

(17) Indicates statements which can be derived

from given information (5)

(18) Indicates if additional data is needed in order to draw a conclusion from a given set of definitions and assumptions (1)

D-2 The student uses mathematical concepts and processes to discover new applications.

The student:

(4) States differences between concepts (1)

(7) Looks for alternative methods of doing problems (2)

(8) Experiments with mathematical possibilities (1)

(9) Makes mathematical conjectures (2)

E-1 The student develops study habits essential for independent progress in mathematics.

The student:

(2) Uses new symbols when necessary (2)

(9) Indicates recognition of patterns and relationships (2)

(10) Changes approach when necessary (1)

(13) Organizes data (5)

(16) Plausibly estimates (3)

F-1 The student is able to continue further work in mathematics.

The student:

(6) Uses a good mathematics vocabulary (11)

F-2 The student is able to continue further work in disciplines which require extensive use of mathematics.

The student:

(1) Capably uses the mathematics associated with future physics courses (5)

(2) Capably uses the mathematics associated with future chemistry courses (2)

(3) Capably uses the mathematics associated with future biology courses (2)

(4) Capably uses mathematics associated with future courses in the social sciences (7)

(5) Capably uses mathematics associated with future astronomy courses (1)

G-1 The student thinks mathematically.

The student:

(3) Matches general statements with related specific ones (7)

(5) Uses correct ideas of implications (1)

(15) Makes accurate and well founded conjectures (1)

G-2 The student understands "good" mathematics.

The student:

(3) Uses algebraic aspects of structure (12)

(4) Applies order aspects of mathematics (1)

(13) Uses the idea of equivalence (1)

(22) Indicates recognition of patterns (1)

H-1 The student is able to critically examine mathematical material.

The student:

(7) Indicates recognition of patterns and relationships (1)

H-2 The student thinks logically.

The student:

(2) Matches general statements with related specific ones (5)

(10) Makes accurate and well founded conjectures (2)

(15) Tells the value of applying the logical thought of mathematics to non-mathematical situations (1)

This completes the listing of those behaviors that were needed by one through fifteen problem solutions. Remaining are those behaviors that were not required by any of the fifty-three problem solutions. This list is the longest as can be seen by once again checking Table III. There are 203 behavioral statements not required by any problem solutions. Because this list is so large, these behavioral statements were put into groups with each group

having some common idea or thought. In each case this common idea is presented and the quantity of behavioral statements that had this idea in common is given as a numeral following the general statement.

1. States the differences, processes, facts, principles, and so on. In general, the student is to state something. (30)
2. Classifies processes, facts, concepts, items, proofs, and general statements. (12)
3. Corrects wrongly stated applications, facts, concepts, references, vocabulary definitions and processes. (11)
4. Tells which characteristics belong to a certain process, if additional data is required, value of applying logical thought. In general, tells about something. (14)
5. Contrasts differences between facts. (1)
6. Identifies situations, assumptions, and conclusions. (2)
7. Reviews a group of situations and indicates those which have a certain common aspect. (3)
8. Describes the role of mathematics in technology, industry, science, further progress, and philosophy. (6)
9. Discusses the importance of mathematics in history,

current affairs, the professions and the trades. (2)

10. Interprets the word "elegance" as it applies to the beauty of mathematics. (1)
11. List practical values of mathematics. (1)
12. Describes the relationship of, traces, names common aspects of, satisfies definition of, uses, and applies vocabulary words. (8)
13. Explains and states the material used to develop algorithms. (2)
14. Indicates; errors in inaccurate solutions, common processes used, type of reasoning used, and untrue implications. (8)
15. Understandably reads problems involving insufficient data, induction or deduction. (3)
16. Analyzes, indicates when complete, constructs, critically evaluates and in general answers questions concerning proofs. (20)
17. Makes statements about the correctness of given solutions with respect to the processes used. (1)
18. Criticizes arguments which make use of facts, concepts, processes, and structure aspects. (5)
19. Traces processes, concepts, and vocabulary items. (4)
20. Indicates common concepts used in two or more

situations. (2)

21. Determines statements as true or false with respect to given information. (3)
22. Looks for counterexamples when this is called for. (3)
23. Changes approach when necessary. (1)
24. Shows an understanding of the structure of mathematics and the use of axioms and postulates. (10)
25. Validates identities. (4)
26. Correctly answers questions which are implications of a theorem presented to him. (7)
27. Speculates about unknown areas. (2)
28. Finds alternative methods, experiments. (3)
29. Attempts to be his own teacher and makes use of intuition. (5)
30. Uses ideas of necessary and sufficient conditions. (5)
31. Explains logical connections between items and relates information to larger whole. (8)
32. Exhibits perseverance. (1)
33. Exhibits work consistent with the next course in mathematics. (4)
34. Indicates an understanding of the qualities of data. (1)
35. Thinks and argues about mathematics for himself. (1)

36. Performs well in future science courses. (1)

37. Makes well founded generalizations. (5)

38. Solves problems which involve insight. (1)

39. Uses correct ideas of implications. (2)

This completes the final grouping of the 360 behavioral statements. These behaviors were put into four groups. In the first group are the forty-six behavioral statements that were required for forty or more problem solutions. The second group contains the thirty-four behavioral statements that were required for twenty-five through thirty-nine behavioral statements. It was found that there were no problem solutions that were required by sixteen through twenty-four problem solutions so these numbers were not included in any group. Those behavioral statements required for one through fifteen problem solutions are in the third group with the total being seventy-seven. In the final group there are 203 behavioral statements with the behaviors in this group being required for none of the problem solutions.

CHAPTER III

SUMMARY AND CONCLUSIONS

I. SUMMARY

The writer first selected the quantitative section of the Iowa Tests of Educational Development as the test to be evaluated. Each of the fifty-three questions were solved with all operations shown on a worksheet. If more than one method could be used, the other methods were also shown.

A list of 360 behavioral statements that should be the goals of secondary mathematics were suggested in a thesis written by E. M. Oltrogge. A checklist was developed that was numbered in the same manner as the list of behavioral statements. The writer then compared a problem solution with each of the 360 behaviors, placing a check-mark by the number of any behavior that was required in the solution of a given problem. This procedure was followed for each of the fifty-three problem solutions.

Using the fifty-three separate checklists, a master checklist was used to record the total number of times that a particular behavior was required. This master checklist was used to group the behaviors into four quantity groups. Those groups are: (1) Those behaviors required forty or more times; (2) Those behaviors required twenty-five through

thirty-nine times; (3) Those behaviors required one through fifteen times; and (4) Those behaviors not required at all. There were no behaviors that were required for sixteen through twenty-four problem solutions.

It was found the first group contained forty-six behaviors, the second thirty-four behaviors, the third seventy-seven behaviors and the last 203 behaviors.

II. CONCLUSIONS

The first conclusion that can be made is that the total number of behaviors that were not required by any problem solutions is quite large. If the number of behaviors required by only one problem solution, thirty-five, were added to the 203 not required by any, this gives a total of 238. This means that 238 of the 360 behaviors, approximately sixty-six per cent, of the behaviors were required only once or not at all. Considering that the author checked a behavior when there was an opportunity to exhibit that behavior, even though a student may be able to solve the problem correctly without exhibiting that behavior, the number of behaviors required for this test was very small.

All behaviors required by forty or more problem solutions can be grouped into the following behavioral statements. Those behaviors grouped are identical in

wording or in thought.

1. The student applies facts, concepts, and processes to problems and new situations which required their use.
2. Uses parts of processes
3. Uses facts to solve problems in everyday life
4. Computes with speed
5. Uses algorithms
6. Uses numerical processes
7. Indicates the plausibility of solutions
8. Selects the correct algorithms
9. Indicates when an answer has been obtained
10. Analyzes problem situations and uses the appropriate processes
11. Solves problems correctly
12. Computes accurately
13. Computes efficiently
14. Completes large numbers of problems in a limited amount of time
15. Understandably reads problems involving transfer
16. Transforms problems into an accessible formulation of the problem
17. Uses necessary concepts to check or verify probable solutions
18. Solves the exact problem stated

19. Is resourceful in problem solving
20. Applies mathematical judgment

These behaviors in general all refer to solving some kind of problem. The student is to: read the problem, select the correct algorithms, compute the answer, and check to see if the answer is correct. The large number of check-marks received by these behaviors points up that a great number of the fifty-three problems must be of this type.

The behaviors that were required for twenty-five through thirty-nine problem solutions can be grouped as follows:

1. Uses processes to check and verify answers
2. Uses processes, facts, and concepts in solving problems of a related field, whenever this is possible
3. Uses mathematics vocabulary words in problems which require their use
4. Interprets solutions
5. Uses processes which are most effective and least wasteful of time
6. Understandably reads problems involving induction or deduction
7. Understandably reads problems involving transfer
8. Solves exact problem stated
9. Demonstrates mathematical reading accuracy

10. Reads with comprehension

11. Uses elementary aspects of number theory

This group of behaviors that also received a very adequate number of checkmarks, has some things in common with the previous group and adds somewhat to previous statements about the first group. In fact, all the behaviors listed except number three would fit into the problem solving type that was described following the first list. Number three listed above involves the use of vocabulary words in problems and the understanding of these vocabulary words. This statement would also fit the general problem solving type question because a student would need to read with comprehension before starting to solve a problem.

The third group of behaviors were those required in one through fifteen problem solutions. Most, sixty-three, of the seventy-seven behaviors were required for five or less problem solutions. This would indicate that these behaviors were not covered very well by the fifty-three problems. The following behaviors are the result of grouping the seventy-seven in the original list. They are:

1. Applies process, facts, and concepts in making predictions or estimates
2. Reviews situations and tells those in which a certain process or concept can be used

3. Indicates true or false items which are based on a concept
4. Tells the value of applying the logical thought of mathematics to non-mathematical situations arising in society
5. Answers simple questions about given vocabulary items
6. Indicates knowledge of arithmetic insights
7. States relationships between numbers and facts
8. Uses new symbols when necessary
9. Corrects wrong computations
10. Performs motor skills satisfactorily
11. Understandably reads problems involving induction, deduction, extrapolation, and insight
12. Makes well founded generalizations
13. Contrasts processes
14. States the facts
15. Makes statements about the correctness of given solutions which make use of a given set of facts
16. Selects suitable notation and symbolism for a given technique
17. Indicates when new ideas are needed
18. Indicates recognition of patterns
19. Answers simple questions about structure
20. Applies aspects of structure to problems and to check solutions

21. Indicates statements which can be derived from given information
22. Uses inequalities and applies order aspects of mathematics
23. Indicates if additional data are needed
24. Organizes details which are related to a given item
25. Eliminates items unrelated to a given topic
26. Changes approach when necessary
27. Capably uses the mathematics associated with future courses in the sciences and social sciences
28. Uses the correct idea of implication
29. Uses the idea of equivalence

Finally, those 203 behaviors that were not covered by any problem solution are in the last list. These behaviors are the most important at this point because they point up the type of mathematical goals that are not covered by this test. These behaviors were grouped earlier on pages sixty-seven through seventy into thirty-nine statements. These statements will not be repeated here but instead some conclusions about this group of behaviors will be stated.

First, it must be remembered that this is a sixty-five minute test and not a complete evaluation of the mathematical ability of the students taking the test. Due to this fact, there are other types of tests and other types of evaluation that would be necessary to test for all 360

behaviors.

The type of behavior as "performs well in future science courses" and "exhibits work consistent with the next course in mathematics" can not be included in this type of test but would need to be observed by someone or a follow-up study should be made. The behaviors "exhibits perseverance" and "thinks and argues about mathematics for himself" are other behaviors that would need to be observed by the teacher of the student.

Some behaviors could be tested more easily by using an essay test than the objective type. An example of this type of behavior is "states the differences, processes, facts, and principles" and "explains and states the material used to develop algorithms." Other examples are "criticizes arguments which make use of facts, concepts, and processes," and "tells which characteristics belong to a certain process."

For the behaviors grouped in number fourteen on page sixty-eight, a question that gave a step-by-step solution with an error at some point in the solution would test for this type of behavior.

To test for the behavior "corrects wrongly stated applications, facts, and concepts" the evaluation instrument would need to give statements that contained concepts or applications and let the student determine if there were

any wrongly stated information and correct the errors that were found.

An evaluation instrument with a choice of "insufficient data" as one of the choices would test for the behavior "understandably reads problems involving insufficient data." Not all problems would need to have this choice but at least one or two would have this as the correct choice.

For the type of behavior "finds alternative methods" and "changes approach when necessary," show a problem and one possible procedure. The student is directed to then show another procedure for solving this problem.

There are eight behaviors that were grouped under the general behavioral statement "describes the relationship of, traces, names common aspects of, satisfies definition of, uses, and applies vocabulary words." This item is very important at this time as the mathematical vocabulary is stressed in the classroom. Vocabulary is also important due to the fact that the mathematics vocabulary of "modern" mathematics is different than that of "traditional." Matching vocabulary items to a list of definitions is one of the several ways of testing for understanding of vocabulary items.

In the last area to be discussed, there are eight general statements that were listed on pages sixty-seven through seventy that are typical of what is being taught in

"modern" mathematics at this time. These general behaviors are listed using the same numbering system as on pages sixty-seven through seventy:

16. Analyzes, indicates when complete, constructs, critically evaluates and in general answers questions concerning proofs. (20)
22. Looks for counterexamples when this is called for. (3)
24. Shows an understanding of the structure of mathematics and the use of axioms and postulates. (10)
25. Validates identities. (4)
26. Correctly answers questions which are implications of a theorem presented to him. (7)
30. Uses ideas of necessary and sufficient conditions. (5)
31. Explains logical connections between items and relates information to larger whole. (8)
37. Makes well founded generalizations. (5)

In this list a total of sixty-two simple behavioral statements are found. This means that almost one-third of those behaviors receiving no checkmarks are included in this list. What type of behaviors are found in this list? Those behaviors that involve the understanding and use of proofs, axioms, and theorems are part of the group. What is found here are behaviors that deal with the structure of

mathematics as a whole. Modern mathematics is being taught so that the student can understand the structure of mathematics rather than the memorizing of a procedure that will solve a problem.

There must be a great many ways that a test writer could include most of these sixty-two behaviors. To evaluate a proof, a mis-structured proof could be given and the student asked to indicate which step or statement is out of place.

The following is an example from the Twenty-Sixth Yearbook of The National Council of Teachers of Mathematics. This problem is given as an example of the behaviors "the student understands the logical structure of mathematics" and "applies mathematical reasoning to non-mathematical situations."

> During a survey in a certain school it was found that some of the students cheat on exams. Assuming the conclusion to be valid, what can be said of the following inferences: (True-False-Indeterminate)?
>
> 1. Some students in the school cheat on exams.
> 2. Many students in the school cheat on exams.
> 3. All students in the school cheat on exams.
> 4. Some students in the school do not cheat on exams.
> 5. Most students in the school do not cheat on exams.
> 6. Some students who do not cheat on exams are in this school.[1]

---

[1]The National Council of Teachers of Mathematics, Evaluation in Mathematics, Twenty-sixth Yearbook (Washington, D.C., 1961), p. 81.

The above examples indicate that there are many areas that are not covered by the Iowa Tests of Educational Development. A total of 203 behaviors were not required for any problem solutions, and thirty-five more were required for only one problem solution. These 238 behaviors comprise approximately sixty-six per cent of all the behaviors listed.

Those behaviors that were required by an adequate number of problems in the test were behaviors required for problem solving. To be a valid test of the goals of secondary mathematics, this test would need to include different types of questions to cover a much larger number of behaviors. The test in present form is inadequate in testing for the goals of modern mathematics.

BIBLIOGRAPHY

BIBLIOGRAPHY

A. BOOKS

Ahmann, J. Stanley, and Marvin D. Glock. Evaluating Pupil Growth. Boston: Allyn and Bacon, Inc., 1963.

Downie, N. M. Fundamentals of Measurement; Techniques and Practices. New York: Oxford University Press, 1967.

Ebel, Robert L. Educational and Psychological Measurement. Edited by David A. Payne and Robert F. McMorris. Waltham, Massachusetts: Blaisdell Publishing Company, 1965.

French, John W., William B. Michael and others. Standards for Educational and Psychological Tests and Manuals. Washington, D. C.: American Psychological Association, Inc., 1966.

Gronlund, Norman E. Measurement and Evaluation in Teaching. New York: Macmillan Company, 1965.

How to Use the Test Results of the Iowa Tests of Educational Development, A Manual for Teachers and Counselors. Iowa City, Iowa: State University of Iowa, 1967.

Lien, Arnold J. Measurement and Evaluation of Learning; A Handbook for Teachers. Dubuque, Iowa: C. Brown Company, 1967.

Lindvall, C. M. Testing and Evaluation; An Introduction. New York: Harcourt, Brace, and World, Inc., 1961.

Magnusson, David. Test Theory. Reading, Massachusetts: Addison-Wesley Publishing Company, 1967.

National Council of Teachers of Mathematics. Evaluation In Mathematics. Twenty-Sixth Yearbook of the National Council of Teachers of Mathematics. Washington: The National Council of Teachers of Mathematics, 1961.

Oltrogge, Eugene M. "Behavioral Objectives for Testing the Relative Merits of Modern and Traditional Mathematics in High School." Master's thesis at Drake University, June, 1965.

Payne, David A. and Robert F. McMorris (ed.). Educational and Psychological Measurement. Waltham, Massachusetts: Blaisdell Publishing Company, 1967.

Tiegs, Ernest W. Tests and Measurements for Teachers. Boston: Houghton Mifflin Company, 1931.

Tyler, Leona E. Test and Measurements. Englewood Cliffs, New Jersey: Prentice Hall Inc., 1963.

Tyler, Ralph W. Constructing Achievement Tests. Columbus, Ohio: Ohio State University, 1934.

B. PERIODICALS

Engelhart, Max D. "What to Look for in a Review of an Achievement Test," The Personnel and Guidance Journal, XLII (February, 1964), 616-619.

Gibbs, Sam M. "New Math Curriculum Changes," The Texas Outlook, IIL (April, 1964), 34-35.

Mueller, Francis J. "The Revolution at Sputnik-Plus-Ten," The Mathematics Teacher, LX (November, 1967), 696-706.

Payne, Holland. "What About Modern Programs in Mathematics," The Mathematics Teacher, LVIII (May, 1965), 422-424.

C. LETTERS

Feldt, Leonard S. Personal letter received by the author. October, 1968.

Williams, Jack R. Personal letter received by the author. November, 1967

APPENDIX

The following problems are from the Iowa Tests of Educational Development, Test 4, "Ability to do Quantitative Thinking." The problem solutions following each problem were used to determine which behaviors were required for a problem solution. The problems are numbered in the same manner as they are in the test booklet. The answer selected as the correct solution is underlined.

1. Henry makes a profit of 40% on each $1.50 box of Christmas cards that he sells. How many boxes must he sell to earn a total profit of $30.00?

(A) 20; (B) 50; (C) 60; (D) 75; (E) Not given

40% = .40

$$\begin{array}{r} \$1.50 \\ \times \quad .40 \\ \hline .6000 \end{array} = \$.60 \text{ per box}$$

$$\frac{\$30.00}{\$.60} = \frac{3000}{60}$$

$$60\overline{)3000}\quad 50$$

therefore 50 boxes are needed.

Check:

$$\begin{array}{r} \$1.50 \\ \times \quad 50 \\ \hline \$75.00 \end{array} \qquad \begin{array}{r} \$75 \\ \times \ .40 \\ \hline \$30.00 \end{array}$$

2. If a piece of pipe 28 3/8 inches long is cut from a pipe 57 1/8 inches long, how long is the piece remaining?

(A) 28 1/4 inches; (B) 29 1/4 inches; (C) 29 3/4 inches;

(D) 29 1/2 inches; (E) Not given

$$\begin{array}{r} 57\ 1/8 \\ -\ 28\ 3/8 \\ \hline \end{array} = \begin{array}{l} 56 + 8/8 + 1/8 \\ -\ 28 + 3/8 \\ \hline \end{array} = \begin{array}{r} 56 + 9/8 \\ -\ 28 + 3/8 \\ \hline 28 + 6/8 \end{array} = 28\ 3/4$$

Check:

$$\begin{array}{r} 28\ 3/8 \\ +\ 28\ 3/4 \\ \hline \end{array} = \begin{array}{r} 28\ 3/8 \\ +\ 28\ 6/8 \\ \hline 56\ 9/8 \end{array} = 56 + 8/8 + 1/8 = 57\ 1/8$$

3. "Four thousand and two hundred twenty-nine thousandths" is the same as:

(A) 0.4229; (B) 4000.229; (C) 4200.029; (D) 4229.000;

(E) Not given

The student must have the ability to read decimal fractions correctly.

The following table gives the difference between 1956 and 1957 sales figures for a retail store during the first six months of the year.

1957 SALES (Thousands of Dollars)
(+ means higher than 1956)
(- means lower than 1956)

MONTH	THOUSANDS OF DOLLARS
January	+ 12
February	+ 4
March	- 10
April	+ 8
May	- 17

4. How do the total sales for 1957 compare with those for 1956 over the entire period?

(A) $51,000 higher than 1956; (B) $11,000 higher than 1956;

(C) Exactly equal to 1956; (D) $3,000 lower than 1956;

(E) Not given

(+12) + (+4) + (-10) + (+8) + (-17) = -3

- 3 x ($1000) = - $3,000 or $3,000 lower.

5. In a large high school graduating class, 129 out of 344 graduates entered college. What per cent of the class entered college?

(A) 26 2/3%; (B) 33 1/3%; (C) 39 1/2%; (D) 40 1/2%;

(E) Not given

Method 1.
$$344\overline{)129.000} \quad .375 = 37.5\% = 37\ 1/2\%$$
$$\begin{array}{r} 103\ 2 \\ \hline 25\ 80 \\ 24\ 08 \\ \hline 1\ 720 \\ 1\ 720 \\ \hline 0 \end{array}$$

Method 2.
$$\frac{129}{344} = \frac{3}{8}$$
$$8\overline{)3.0000} \quad .375 = 37\ 1/2\%$$
$$\begin{array}{r} 24 \\ \hline 60 \\ 56 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

Method 3. $344r = 129$

$$r = \frac{129}{344}$$

Then divide as above.

Because problems 6 and 7 are based on the same data that is given in the form of a bar graph, problem 8 is given here and problems 6 and 7 are given on the following page.

8. The boiling point of water, which is taken as 212° F. at sea level, decreases by 1 degree for each 511 feet of altitude above sea level. To the nearest degree, what would be the boiling point in Denver, which is at an altitude of 5,280 feet?

(A) 99° F.; (B) 202° F.; (C) 210° F.; (D) 211° F.;

(E) Not given

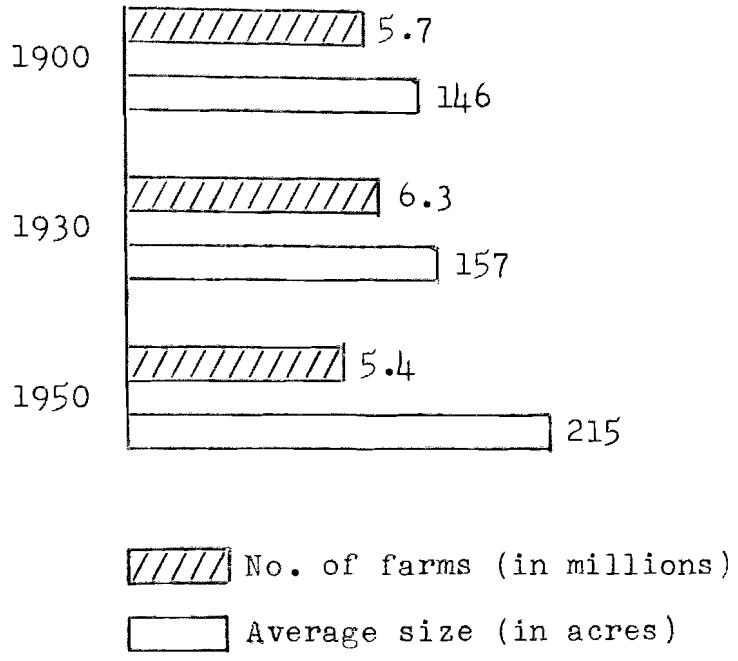
$$511\overline{)5280} \quad 10$$
$$\begin{array}{r} 511 \\ \hline 170 \end{array}$$

10 and 170/511 which is equal to 10 degrees to the nearest degree.

$$\begin{array}{r} 212 \\ -\ 10 \\ \hline 202 \end{array} = 202 \text{ degrees F.}$$

Problems 6-7 are based on the following graph.

How Agriculture Has Changed in the United States



6. How many farms were there in 1950?

(A) 54,000; (B) 540,000; (C) 5,400,000; (D) 54,000,000; (E) 540,000,000

$5.4 \times 1{,}000{,}000 = 5{,}400{,}000$

7. Which of these overall changes has occurred from 1900 to 1950?

(A) Farms have decreased in number and size.

(B) Farms have increased in number and size.

(C) Farms have increased in number and decreased in size.

(D) Farms have decreased in number and increased in size.

(E) Farms have remained unchanged in number and increased in size.

9. On a public works project the city is to pay 1/5 of the cost, the state 1/3 of the cost, and federal government the balance. If the job cost $18,000, what will be the federal government's share?

(A) $7,200; (B) $8,400; (C) $9,600; (D) $13,500;

(E) Not given

Method 1:

$1/5 \times 18{,}000 = 3{,}600$

$1/3 \times 18{,}000 = 6{,}000$

$3{,}600 + 6{,}000 = 9{,}600$

$18{,}000 - 9{,}600 = 8{,}400 = \$8{,}400$

Method 2:

$1/5 = 3/15$

$+ \ 1/3 = 5/15$

$8/15$

$1 - 8/15 = 15/15 - 8/15 = 7/15$

$7/15 \times 18{,}000 = 8{,}400$

10. If a board six feet long is cut into three parts so that the parts are in the ratio of 3:1:1, what would be the length in feet of each of the two shorter pieces?

(A) 10 inches; (B) 2 feet; (C) 2 2/5 feet;

(D) 4 feet; (E) Not given

Let the first part be $3x$
Let the second part be x
Let the third part be x

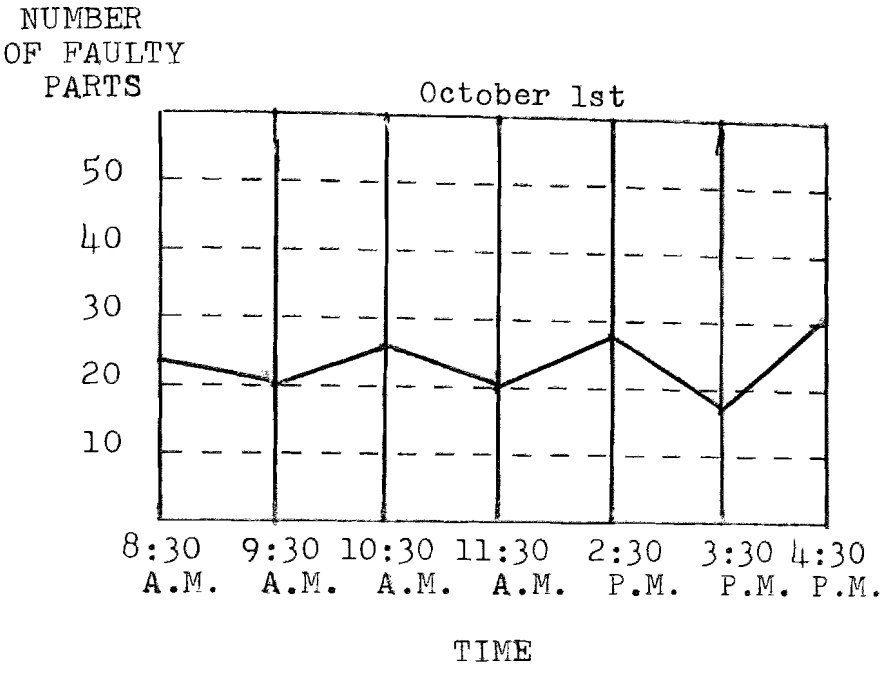
$3x + x + x = 6$

$5x = 6$

$x = 6/5$

The two shorter pieces are each 1 and 1/5 feet.

Problem 11 is based on the following graph



11. In a large factory a sample of 1000 parts is taken out of the production line every hour and inspected for flaws. The chart above gives the number of faulty parts found in the samples for October 1. Which of the following statements best summarizes this information?

(A) Overall production was lower in the afternoon.

(B) Overall production was higher in the afternoon.

(C) Quality was more consistent in the morning.

(D) Fewer faulty parts were produced in the morning.

(E) A smaller proportion of acceptable parts was produced in the afternoon.

12. If there are 2.54 centimeters to the inch, which of the following indicates the correct computation for finding the number of centimeters (c) in P yards?

(A) $c = 12 \times 2.54 \times P$; (B) $c = 12 \times 36 \times 2.54 \times P$;

(C) $c = (36 \times 2.54 \times P) \div 12$; (D) $c = 36 \times 2.54 \times P$;

(E) Not given

2.54 cm. = 1 inch

1 yd. = 36 inches

P yards = (P x 36) inches, therefore $c = 2.54 \times 36 \times P$

13. If $3(x - 1) = 2(x + 1)$, what does x equal?

(A) +5; (B) - 1/5; (C) +1; (D) -1; (E) Not given

Method 1:

$3(x - 1) = 2(x + 1)$	Given
$3x - 3 = 2x + 2$	Distributive Property
$3x = 2x + 5$	Addition of three to both sides of equation
$x = 5$	Addition of $(-2x)$ to both sides of equation

Method 2: Try each solution in the given equation to see if it is the correct answer.

14. A certain raw ore, as it comes from the mine, contains 8% pure metal. How many tons of raw ore must be processed to get one ton of pure material?

(A) 8; (B) 12; (C) 25; (D) 250 (E) Not given

8% = .08

Let n represent the number of tons needed.

.08n = 1 Given

8n = 100 Multiply both sides by 100

n = 12 1/2 Dividing both sides by 8

Check:
$$\begin{array}{r} 12.5 \\ \times\ .08 \\ \hline 1.000 \end{array}$$

Use this table for numbers 15-16.

A lumber yard uses the following table to aid clerks in figuring the cost of oak flooring:

No. of ft.	10	20	30	40
Cost	$.47	$.87	$1.27	$1.67

15. How much should a clerk charge for 55 feet of flooring?

(A) $2.07; (B) $2.27; (C) $2.57; (D) $3.07;

(E) Not given

$$\begin{array}{r} 55 \\ \times\ .04 \\ \hline 2.20 \end{array} \qquad \begin{array}{r} \$2.20 \\ +\ .07 \\ \hline \$2.27 \end{array}$$

16. Which of the following formulas express the correct relationship between F, the number of feet of flooring, and C, the total cost?

(A) C = F + 37; (B) C = 2F + 27; (C) C = 4F + 7;

(D) C = 3F + 17; (E) Not given

17. Gold can be beaten to a thickness of 2×10^{-5} inches. What is another way to write this measurement?

(A) 200,000.0 inches; (B) 20,000.0 inches; (C) 0.00002 inches; (D) 0.000002 inches; (E) Not given

$10^{-5} = \frac{1}{100000} = 0.00001$ $\quad 2 \times 0.00001 = .00002$ inches

18. In 1960 a giant tree was estimated to be 5000 years old. About what year did it first begin to grow?

(A) 3040 B.C.; (B) 3140 B.C.; (C) 4040 B.C.;

(D) 5000 B.C.; (E) 6960 B.C.

$1960 - 5000 = -3040$ $\quad$ -3040 is the same as 3040 B.C.

19. The first step in a convenient method for multiplying large numbers such as 3956 by 298 is to multiply 3956 by 300. The second step would be to:

(A) subtract 2 times 298; (B) subtract 600 from the result;

(C) divide 3956 by 2; (D) subtract 2 times 3956;

(E) add 596 to the result

Method 1:

$3956 \times 298 = 3956(300 - 2) = (3956 \times 300) - (3956 \times 2)$

Method 2:

$300 \times 3956 = 1,186,800$
$298 \times 3956 = 1,178,888$

$$\begin{array}{r} 1,186,800 \\ -\ 1,178,888 \\ \hline 7,912 \end{array}$$

$7,912 = 3956 \times 2$

20. Exactly how many tens are there in the number 571.83?

(A) 57.183; (B) 71.83; (C) 571.83; (D) 5718.3;

(E) Not given

$$10\overline{)571.830}\quad 57.183$$

21. The per capita consumption of meat in the United States was 161 pounds in 1955. Which of the following interpretations of this statement is most accurate?

(A) The typical adult living in the United States ate 161 pounds of meat in 1955.

(B) In 1955 half the people of the United States ate more than 161 pounds of meat, half ate less than 161 pounds.

(C) Most of the people in the United States ate 161 pounds or more of meat in 1955.

(D) If all the people in the United States in 1955 had eaten an equal amount of meat, each would have had 161 pounds.

(E) All but an insignificant proportion of the people in the United States consumed at least 161 pounds of meat in 1955.

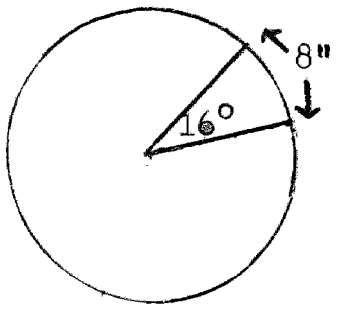
22. A rectangular corn crib with inside dimensions of 10 feet by 20 feet is filled to a level 14 feet high with 3500 bushels of corn. How many bushels are represented by each one foot of height in the crib?

(A) 25 bushels; (B) 17.5 bushels; (C) 125 bushels;

(D) 250 bushels; (E) Not given

$$\begin{array}{r} 250 \\ 14\overline{)3500} \\ \underline{28} \\ 70 \\ \underline{70} \\ 0 \end{array}$$

Problem 23 is based on the following figure:



23. The length in inches of the circumference of the at circle is:

(A) 90 inches; (B) 128 inches; (C) 160 inches;

(D) 180 inches; (E) Not given

Method 1:

$$\frac{16^\circ}{360^\circ} = \frac{8''}{x}$$

$$16x = 2880$$

$$x = 180 \text{ inches}$$

Method 2:

$$16\overline{)8.0} \quad .5 \text{ inches per 1 degree}$$

$$\begin{array}{r} 80 \\ 0 \end{array}$$

$$\begin{array}{r} 360^\circ \\ \times \quad .5 \text{ inch} \\ \hline 180.0 \text{ inches} \end{array}$$

24. Mr. Rogers drove his car 214 miles on 11.2 gallons gasoline costing 28.8¢ per gallon. From these data, wha would the quantity $\frac{11.2 \times 28.8}{214}$ tell us?

(A) Cents per mile; (B) Miles per dollar; (C) Gallo per mile; (D) Dollars per mile; (E) Not given

$$\frac{11.2 \text{ gallons}}{1} \times \frac{28.8 \text{ cents}}{1 \text{ gallon}} \times \frac{1}{214 \text{ miles}} = \frac{\text{cents}}{\text{mile}}$$

25. What is the next term of the series 1, 9, 25, 49?

(A) 73 (B) 81; (C) 89; (D) 97; (E) Not given

Method 1:

$1^2 = 1$

$3^2 = 9$

$5^2 = 25$

$7^2 = 49$

$9^2 = 81$ Therefore 81 is the number

Method 2:

The number	Its square root
1	1
9	3
25	5
49	7

The next square root appears to be 9 because the square roots are odd numbers in order. If 9 is the next square root, then 81 is number needed because 9 is the square root of 81.

26. Which of these sets of data would be most appropriately presented by a circle graph?

(A) Safety records of ten drivers working for a trucking company

(B) Changes in the cost of ten basic food items from 1945 to 1955

(C) Worker productivity in the steel industry in five major countries

(D) Population growth in the United States at twenty-year intervals from 1800 to 1950

(E) Percentage breakdown of the profits of a company from its various products

27. In an experiment on the productivity of four varieties of tomato plants, the yield from ten plants of each kind was weighed and tabulated. The results are given below:

YIELD	No. of plants of each variety giving the indicated yield			
	Variety A	Variety B	Variety C	Variety D
5 pounds	1	1		
4 pounds	2	5	1	1
3 pounds	4	3	3	7
2 pounds	2	1	4	2
1 pound	1		2	
Total no. of plants	10	10	10	10

On the basis of this experiment, which variety should be recommended?

(A) A; (B) B; (C) C; (D) D; (E) No variety was superior to any other.

Variety A	Variety B	Variety C	Variety D
5	5	4	4
8	20	9	21
12	9	8	+ 4
4	+ 2	+ 2	29 lbs.
+ 1	36 lbs.	23 lbs.	
30 lbs.			

28. If a student ranks 23rd from the top in a class of 187, which of the following most precisely and accurately places his position in the class?

(A) Within the top 5%; (B) Within the top 10%;

(C) Within the top 25%; (D) Within the top one-third;

(E) Within the top one-fifth

$$187\overline{)23.000} \quad .123 = 12.3\%$$

29. If a pound of grass seed is sufficient for 250 square feet, how many pounds would be required for the lawn diagrammed below?

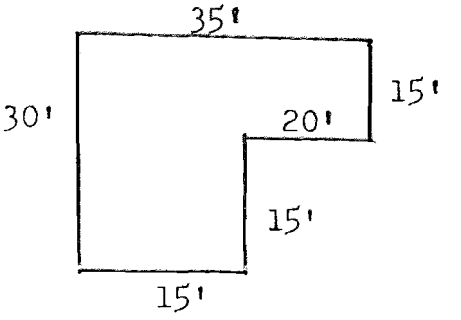
(A) 3.0

(B) 3.9

(C) 4.2

(D) 5.1

(E) Not given



The student must know how to find the area of a rectangle.

35 x 30 = 1050 sq. ft. which is the total area if part of rectangle were not missing.

15 x 20 = 300 sq. ft. which is the area of the part that appears to be removed from the bottom.

1050 - 300 = 750 square feet which is the area of the diagram.

$$250\overline{)750}\quad 3 \quad = 3.0 \text{ lbs. of grass seed}$$
$$\underline{750}$$
$$0$$

30. A man is hired at an annual salary of $4800 with annual increases of $200, effective after each full year with the company. Which of the following expressions gives his salary after 'Y' years?

(A) 200Y + 4800; (B) 200(Y + 4800); (C) 4800(200 + Y);

(D) 4800(200)(Y); (E) Not given

The student must understand a(b + c) = (ab) + (ac)

31. Radioactive materials undergo continuous disintegration at rates that have been accurately calculated. The amount (A) of one such material that remains after Y years is defined by the relationship $A = 800(\frac{1}{2})^{Y/25}$

What amount will remain after fifty years?

(A) 200 units; (B) 400 units; (C) 600 units;

(D) 800 units; (E) Not given

Y = 50, therefore Y/25 is equal to 2. The relationship then becomes $A = 800(\frac{1}{2})^2$

$$A = 800(\tfrac{1}{4})$$

$$A = 200$$

Because problems 32 and 33 use the same table, these problems are on the following page.

34. Mr. Hudson drove his car 8,470 miles during a recent year. In making out his income tax, he wished to list the money that he paid in federal gasoline taxes as a deduction. If Mr. Hudson averages 14 miles to the gallon and the federal tax is 6¢ per gallon, what is the total amount of this deduction?

(A) $36.30; (B) $39.00; (C) $50.82; (D) $71.15;

(E) Not given

$$14\overline{)8470} = 605 \text{ gallons of gasoline}$$

$$\begin{array}{r} 84 \\ \hline 07 \\ 00 \\ \hline 70 \\ 70 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 605 \\ \times\ .06 \\ \hline \$36.30 \end{array}$$

Problems 32 and 33 are based on the following table.

Population of the United States by Age: 1920 - 1960

(Figures Given in Hundred Thousands)

Age	1920	1930	1940	1950	1960 (Predicted)
Under 20	430	476	453	511	671
20 - 39	343	390	425	464	466
40 - 59	203	257	302	348	410
60 and over	80	105	137	184	233
Total	1056	1228	1317	1507	1780

32. What increase was expected between 1950 and 1960 in the number of people 60 and over?

(A) Forty-nine thousand

(B) Four hundred ninety thousand

(C) Four million, nine hundred thousand

(D) Forty-nine million

(E) Not given

$233 - 184 = 49$

$49 \times 100{,}000 = 4{,}900{,}000$

33. What was the first of the tabled years in which the group between 20 and 60 included less than half of the total population?

(A) 1920; (B) 1930; (C) 1940; (D) 1950; (E) 1960

343 + 203 = 546	390 + 257 = 647	425 + 302 = 727	464 + 348 = 812	466 + 410 = 876
546 x 2 = 1092	647 x 2 = 1294	727 x 2 = 1454	812 x 2 = 1624	876 x 2 = 1752

Each product was compared with the totals in the table.

35. Which of the following means the same as $r + r + r + r$?

(A) $r + 3$; (B) $4r$; (C) $2r^2$; (D) r^4 (E) Not given

The student must understand the use of exponents and perhaps $r + r + r + r = r(1 + 1 + 1 + 1) = r.4 = 4r$

36. According to historical records, the life expectancy of men born in 1843 was 41 years. What does this mean?

(A) More men died at 41 than at any other age.

(B) The average age at time of death was 41.

(C) Very few men lived to be 41.

(D) All but an insignificant proportion lived to be 41 or older.

(E) No correct interpretation is given.

37. A store manager made the following deposits during one week of business: Monday, \$135.65; Tuesday, \$107.83; Wednesday, \$93.24; Thursday, \$125.98; Friday, \$268.15; Saturday, \$255.75. During this week withdrawals of \$145.38 and \$251.46 were made. How much had the account increased by the end of the week?

(A) \$490.56; (B) \$589.86; (C) \$590.66; (D) \$986.60;

(E) Not given

\$135.65
107.83
93.24
125.98
268.15
+ 255.75
\$986.60

\$145.38
+ 251.46
\$396.84

\$986.60
- 396.84
\$589.76

38. As a rule of thumb in planning the size of rooms in a new building, architects usually allow a minimum of 200 cubic feet for each occupant. By this rule, which of the following floor dimensions would result in the smallest acceptable meeting room accommodating 30 persons, assuming a ceiling height of 10 feet?

(A) 15 x 20; (B) 20 x 25; (C) 20 x 30; (D) 25 x 30;

(E) None of these would be acceptable.

200 x 30 = 6000 cubic feet needed

10 x 15 x 20 = 3000 cubic feet
10 x 20 x 25 = 5000 cubic feet
10 x 20 x 30 = 6000 cubic feet
10 x 25 x 30 = 7500 cubic feet

Students must know the formula for volume of a rectangular solid or understand the procedure for finding volume.

39. In a circle graph showing the expenditures for the federal government, the segment representing the amount spent for military operations has an angle at the center of the circle equal to 190 degrees. According to this graph, approximately how many cents out of every dollar spent by the federal government goes for military operations?

(A) 19¢; (B) 47¢; (C) 50¢; (D) 53¢; (D) 89¢

$$\frac{190}{360} = \frac{x}{100}$$

$$19000 = 360x$$

$$53 = x$$

$$360\overline{)19000} = 52\ 280/360 \text{ or } 52\ 7/9$$

$$\begin{array}{r} 52 \\ 360\overline{)19000} \\ \underline{1800} \\ 1000 \\ \underline{720} \\ 280 \end{array}$$

The student must know the central angle of a circle contains 360 degrees.

40. Which of these expressions correctly represents the statement, "S equals the sum of the squares of the numbers x, y, and z"?

(A) $S = (x + y + z)^2$; (B) $S = x^2 + y^2 + z^2$;

(C) $S = 2x + 2y + 2z$; (D) All of the above expressions are correct. (E) None of the above expressions is correct.

41. The following information appeared on a worker's pay envelope:

Total Wages:	\$65.00
Witholding Tax:	14.30
Social Security:	.65
Net Wages:	\$50.05

The worker's "take-home pay" was what per cent of his total wages?

(A) 77%; (B) 72%; (C) 55%; (D) 28%; (E) Not given

$$65\overline{)50.05} = .77$$

$.77 = 77\%$

$x\%$ of $65 = 50.05$

$65x = 50.05$

$x = .77$

42. A watch gains 2 1/2 minutes an hour. It is set accurately at 10:00 P. M. How many minutes will the watch have gained by 6:30 A.M. the following morning?

(A) 16 1/4; (B) 18 3/4; (C) 20; (D) 41 1/4

(E) Not given

10:00 P.M. to 12:00 midnight is 2 hours $2 + 6\ 1/2 = 8\ 1/2$
12:00 to 6:30 A.M. is 6 1/2 hours

$(8\ 1/2) \times (2\ 1/2) = 85/4$ or 21 1/4 minutes.

43. If the temperature at 11:00 A.M. was 15° and at 4:00 P.M. the same day it was -5 degrees, what was the average change per hour during this period?

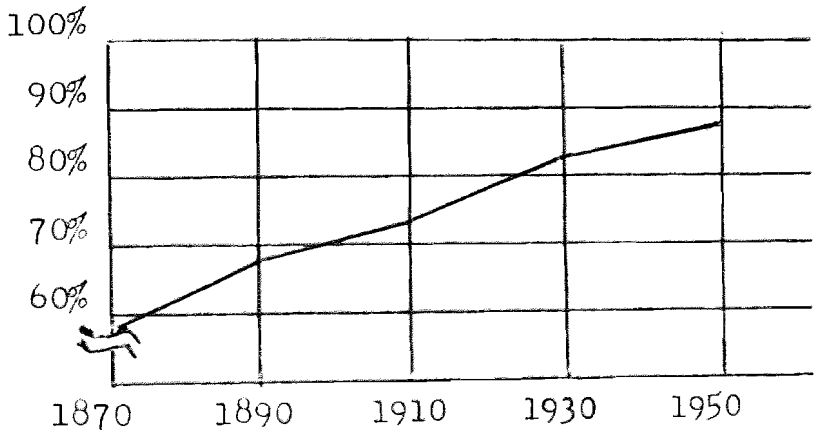
(A) $+5^{\circ}$; (B) $+2^{\circ}$; (C) -2°; (D) -4°; (E) Not given

$15 - (-5) = 20$ degrees change downward

$\frac{-20}{5} = -4^{\circ}$

Problem 44 is based on the following graph.

Per cent of Children Aged 5-17 Attending School: 1870 - 1950



44. Assuming there were 30 million children between the ages 5 and 17 in 1950, approximately how many children were not attending school?

(A) 250,000; (B) 500,000; (C) 2,500,000; (D) 5,000,000;

(E) 25,000,000

100% - 85% = 15%

30,000,000 x (.15) = 4,500,000 or approximately 5,000,000.

45. If a blueberry pancake recipe which makes 4 servings calls for 1 3/4 cups of pancake mix, how much mix should be used if 6 servings are wanted?

(A) 2 1/4 cups; (B) 2 3/8 cups; (C) 2 1/2 cups;

(D) 2 5/8 cups; (E) Not given

$\frac{4}{1\ 3/4} = \frac{6}{x}$ if x represents the cups of mix

$4x = (6)(1\ 3/4)$

$4x = 10\ 1/2$

$x = 2\ 5/8$

Problem 46 is based on the data below.

DAILY NEWS WANT AD RATES

7 consecutive days	48¢ per line per day
3, 4, 5, 6 consecutive weekdays	51¢ per line per day
One day (weekdays)	60¢ per line
Sunday	80¢ per line
Minimum size ad	2 lines

46. What is the price of a three-line ad to appear in the paper from Monday through Thursday?

(A) $1.53; (B) $2.04; (C) $6.12; (D) $7.20;

(E) Not given

$4 \times 51¢ = \$2.04$ $\$2.04 \times 3 = \$6.12$

47. The price of a certain grade of beef cattle is currently 24¢ per pound, and the price has been decreasing approximately .2¢ per pound each week. If this trend continues, when will the price decline to 20¢ per pound?

(A) 2 weeks; (B) 10 weeks; (C) 12 weeks; (D) 200 weeks; (E) Not given

24¢ - 20¢ = 4¢, the total price decline

Let n represent the number of weeks needed

$.2n = 4$

$n = 20$

$.2\overline{)4.0} = 2\overline{)40}$ (20)

48. What is the value of $(.04)^3$?

(A) .12; (B) .64; (C) .000012; (D) .00006$\underline{4}$; (E) Not given

$(.04)^3 = (.04)(.04)(.04)$

$$\begin{array}{r} .04 \\ \times\ .04 \\ \hline .0016 \end{array}$$

The student must know the meaning of exponents.

$$\begin{array}{r} .0016 \\ \times\ .04 \\ \hline .000064 \end{array}$$

49. A salesman's commission is computed on his total dollar volume of sales per month as follows:

10% on the first $400 (or portion thereof)
12% on the next $800 (or portion thereof)
15% on the next $800 (or portion thereof)
20% on all over $2000

If a salesman makes sales totaling $3630 during a given month, what will be his income?

(A) $582.00; (B) $590.00; (C) $626.00; (D) $652.00;

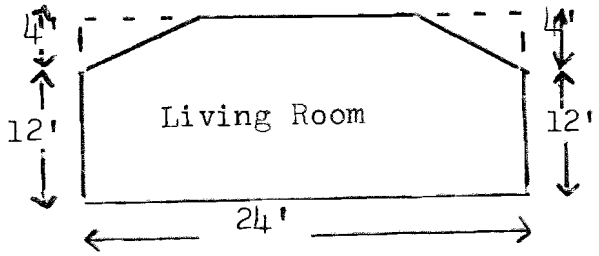
(E) $752.00

$400 x .10 = $40.00
$800 x .12 = $96.00
$800 x .15 = $120.00

$1630 x .20 = $326.00

$3630
- 2000
$1630

$ 40
96
120
+ 326
582



50. Which of the following would give the number of square feet of carpet necessary to cover the above living room floor?

(A) (24 x 16) - (4 x 6); (B) (24 x 12) + (4 x 12);

(C) (24 x 16) - 2(4 x 6); (D) (24 x 12) + 2 $\left[\frac{4 \times 6}{2}\right]$

(E) Not given

Area of total rectangle is (24 x 16)
Area of each triangle that is missing is (1/2)(4 x 6)
Area of two triangles is (2)(1/2)(4 x 6) = 1(4 x6) = (4 x 6)
Total area minus the missing parts equals (24 x 16)-(4 x 6)
The student must know how to find the area of a rectangle and a triangle.

51. The rate of flow through a water pipe is proportional to the square of the diameter. If the flow through one-half-inch is 5 gallons per minute, what will it be through one-inch pipe?

(A) 10 gallons per minute; (B) 20 gallons per minute;

(C) 40 gallons per minute; (D) 80 gallons per minute;

(E) Not given

$\frac{(1/2)^2}{1^2} = \frac{5}{x}$ if x represents the rate of flow through the one-inch pipe

$\frac{(1/4)}{1} = \frac{5}{x}$

$\frac{x}{4} = 5$

$x = 20$

Problems 52 and 53 are on the following page because both are based on the same data.

Problems 52 and 53 are based on the following data: Railroad freight rates are based on the type of merchandise, its weight, and the distance it is hauled. Rates are quoted per 100 pounds; for shipments over 100 pounds, charges are figured on a per-pound basis, using the 100-pound rate as a base. The following table gives the rates on three products:

	100 Miles	200 Miles	300 Miles
Water Heaters	$1.34	$1.74	$2.08
Electric Ranges	$1.56	$2.04	$2.44
Television Sets	$1.71	$2.25	$2.80

52. How much would it cost to ship a 150 pound water heater 200 miles?

(A) $1.54; (B) $1.74; (C) $2.56; (D) $2.61;

(E) Not given

$$100\overline{)150.0} = 1.5 \text{ = number of 100 pounds}$$

100
50 0
50 0
0

$$\$1.74 \times 1.5 = \$2.61$$

53. If the rate for distances between 200 and 300 miles increases in proportion to the distance, what is the charge for shipping a 280 pound electric range 240 miles?

(A) $2.20; (B) $4.03; (C) $6.16; (D) $6.27;

(E) Not given

$\$2.44 - \$2.04 = \$.40$
$240 - 200 = 40$ miles

$$100\overline{).4000} = .004 \text{ = cost per mile}$$

(40 miles) x (.004) =.16
or $0.16 per hundred pounds

$\$2.04 + .16 = \$2.20$ as rate for 240 miles per 100 pounds

$$100\overline{)280.0} = 2.8 \text{ = number of 100 pounds}$$

200
80 0
80 0
0

$$\$2.20 \times 2.8 = \$6.16$$

The following listing shows which behavioral statements were checked for each problem. The numbers listed here refer to the behaviors on pages 16 through 42.

Problem 1.

I. A-1 1
4
A-2 1
12
A-3 1
C-1 1
II. A-1 1
6
7
8
10
11
13
A-2 2
6
7
8
9
10
A-3 1
2
3
5
6
7
9
B-1 8
13
15
IV. A-1 1
A-2 1
A-3 1
B-1 1
3
6
9
12
D-1 4
E-1 1
3
4
5
17
18
F-1 7
F-2 7
8

Problem 2.

I. A-1 1
4
5
A-2 1
5
12
A-3 1
4
II. A-1 1
2
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
4
5
6
7
9
B-1 8
13
15
IV. A-1 1
6
A-2 1
6
A-3 1
6
B-1 1
3
6
9
12
13
17
18
19
F-1 7
F-2 7
8
G-2 5
H-1 10

Problem 3.

I. A-1 1
A-2 1
A-3 1
11
C-1 1
4
II. A-1 8
11
13
A-2 2
6
9
A-3 6
7
8
B-1 1
4
7
8
10
16
17
IV. A-1 1
3
A-2 4
A-3 1
6
B-1 1
12
C-1 1
D-1 4
E-1 1
4
5
17
F-1 7
F-2 7
8
G-1 10
12
G-2 17
19
H-2 6
8

Problem 4.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7
II. A-1 1
6
7
8
10
11
12
13

Problem 4 cont.

II. A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 4
8
10
13
15
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
B-1 1
3
6
9
12
C-1 3
5
D-1 4
E-1 1
3
4
5
17
18
19
F-1 7
F-2 7
8
G-1 12
G-2 3
5
19
H-1 10
H-2 8

Problem 5.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7
II. A-1 1
4
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
4
7
8
B-1 10
13
15
16
17
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
17
18
19
F-1 7
F-2 7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 6

I. A-1 1
4
8

A-2 1
7
12
A-3 1
7
II. A-1 1
6
7
10
11
13
A-2 2
6
7
9
10
A-3 1
2
3
5
6
7
9
B-1 3
8
13
15
IV. A-1 1
A-2 1
10
A-3 1
6
10
B-1 1
3
6
9
12
D-1 4
10

Problem 6 cont.

IV. E-1 1, 3, 4, 5, 17, 18, 19
F-1 7
F-2 7, 8
G-2 5

Problem 7.

I. A-1 1, 8
A-2 1, 7, 12
A-3 1, 7
C-1 1

II. A-1 4, 8, 11, 12, 13
A-2 2, 6, 9, 10
A-3 6, 7, 8, 9
B-1 1, 3, 4, 7, 8, 10, 16, 17

IV. A-1 1
A-2 7, 10
A-3 1, 6, 10
B-1 1, 3, 6, 12
C-1 10
D-1 4, 11
E-1 1, 4, 5, 6, 17
F-1 7
F-2 4, 7, 8
G-1 3, 10, 12
G-2 17, 19, 23
H-2 2, 6, 8

Problem 8.

I. A-1 1, 4, 5, 8
A-2 1, 5, 7, 12
A-3 1, 4, 7
C-1 1

II. A-1 1, 6, 7, 8, 10, 11, 12, 13
A-2 2, 5, 6, 7, 8, 9, 10
A-3 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
B-1 4, 8, 10, 13, 15

IV. A-1 1, 6
A-2 1, 6, 10
A-3 1, 6, 10
B-1 3, 6, 9, 12, 13
D-1 4, 9
E-1 1, 3, 4, 5, 6, 17, 18, 19
F-1 7
F-2 1, 7, 8
G-1 12
G-2 5, 19, 23
H-1 10
H-2 6, 8

Problem 9.

I. A-1 1, 4, 5
A-2 1, 5, 12
A-3 1, 4

II. A-1 1, 6, 7, 8, 10, 11, 12, 13, 15
A-2 2, 4, 6, 7, 8, 9, 10
A-3 1, 2, 3, 5, 6, 7, 8, 9
B-1 8, 13, 15

Problem 9 cont.

IV. A-1 1
A-2 1
A-3 1
6
B-1 1
3
6
9
12
D-1 4
6
D-2 7
E-1 1
3
4
10
17
18
19
F-1 7
F-2 7
8
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
4
7
8
10
13
15
16
17
IV. A-1 1
6
A-2 1
4
10
A-3 1
6
10
B-1 1
3
6
9
12
13
C-1 3
19
D-1 4
11
E-1 1
3
4
5
E-1 6
17
18
19
F-1 7
F-2 7
8
G-1 10
12
G-2 3
5
17
19
23
H-1 10
H-2 6
8

Problem 10.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7
II. A-1 1
6
7
8
10
11
12
13

Problem 11.

I. A-1 1
8
A-2 7
A-3 1
7
B-1 3
II. A-1 11
13
A-2 9
10
A-3 6
7
9
B-1 1
4
8
10
11
16
17
IV. A-1 1
A-2 10
A-3 1
8
10
B-1 1
3
12
C-1 10
D-1 4
11
17
D-2 9
11
E-1 1
4
5
6
17
F-2 7
8
G-1 3
10
12
G-2 19
23
H-2 2
6
8

Problem 12.

I. A-1 1
4
5
A-2 1
5
12
A-3 1
4
II. A-1 1
4
6
7
8
9
10
11
12
13
A-2 2
5
6
8
9
10

Problem 12 cont.
II. A-3 1
2
3
4
5
6
7
8
9
B-1 1
4
6
7
8
10
13
15
16
17
IV. A-1 1
6
A-2 1
6
A-3 1
6
B-1 1
3
6
7
8
9
12
13
C-1 5
D-1 4
11
17
E-1 1
2
3
4
5
6
17
19
F-1 7
F-2 1
7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 13.
I. A-1 1
4
A-2 1
12
A-3 1
4
II. A-1 1
6
7
8
9
10
11
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
9
B-1 8
10
13
15

IV. A-1 1
6
A-2 1
6
A-3 1
6
B-1 1
3
6
7
8
9
C-1 3
5
10
D-1 4
17
E-1 1
3
4
5
17
F-2 7
8
G-2 3
5
13

Problem 14.
I. A-1 1
4
A-2 1
12
A-3 1
II. A-1 1
6
7
8
10
11
12
13

A-2 2
6
7
8
9
10
A-3 1
2
3
4
5
6
7
8
9
B-1 1
4
8
13
15
16
17
IV. A-1 1
A-2 1
A-3 1
B-1 1
3
6
9
12
C-1 3
D-1 4
E-1 1
3
4
5
17
18
19
F-1 7
F-2 7
8
G-1 5
G-2 3

Problem 15.
I. A-1 1
4
5
8
A-2 1
5
6
7
12
A-3 1
7
II. A-1 1
6
7
8
10
11
12
13
A-2 2
3
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
4
5
7
8
10
13
15
16
17

IV. A-1 1
6
A-2 2
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
13
17
18
19
F-1 7
F-2 7
8
G-1 10
12
G-2 17
19
23
H-1 10
H-2 6
8

Problem 16.
I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7

C-1 1
II. A-1 1
4
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
4
7
8
10
13
15
16
17
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
8
9
12
13

D-1 4
11
E-1 1
3
4
5
6
17
18
19
F-1 6
7
F-2 7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 17.
I. A-1 1
A-2 1
7
12
A-3 1
7
II. A-1 1
6
7
9
10
11
13
A-2 2
7
8
9
10
A-3 1
2
3
4
5
6

Problem 17 cont.

II. A-3 7
9
B-1 8
10
15
16
IV. A-1 1
A-2 1
10
A-3 1
10
B-1 1
3
7
9
12
D-1 4
E-1 3
4
5
18
F-1 7
F-2 8

Problem 18.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
7
C-1 1
II. A-1 1
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
6
10
B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
17
18
19
F-1 7
F-2 3
7
F-2 8
G-1 10
12
G-2 17
19
23
H-1 10
H-2 6
8

Problem 19.

I. A-1 1
4
A-2 1
5
6
A-3 1
4
6
C-1 1
II. A-1 1
5
6
7
10
11
13
A-2 2
7
8
9
10
A-3 1
2
3
5
6
7
9
B-1 1
10
IV. A-1 1
A-2 1
9
A-3 6
8
15
B-1 1
3
9
12
C-1 3
5
11
15
D-1 1
4
6
E-1 3
4
5
6
19
F-2 8
G-1 12
G-2 3
H-2 6

Problem 20.

I. A-1 1
4
5
A-2 1
5
A-3 1
4
C-1 1
II. A-1 1
4
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10

Problem 20 cont.
II. A-3 1
2
3
5
6
7
8
9
B-1 1
4
7
8
10
13
15
16
17
IV. A-1 1
6
A-2 1
A-3 1
6
B-1 1
3
6
9
12
13
C-1 1
5
D-1 4
11
E-1 1
3
4
5
6
17
18
19
F-1 6
7
F-2 7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 21.
I. A-1 1
4
8
A-2 1
5
7
12
A-3 1
7
C-1 1
II. A-1 4
8
11
12
13
A-2 9
10
A-3 2
6
7
9
B-1 1
4
8
10
16
17
IV. A-1 1
A-2 1
10
A-3 1
10
B-1 1
3
6
12
C-1 10
D-1 2
4
11
17
D-2 9
E-1 1
4
5
6
17
F-1 6
7
F-2 4
7
8
G-1 3
10
12
G-2 5
19
23
H-2 6
8

Problem 22.
I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 4
7
C-1 1
II. A-1 1
6
7
8
10
11
12
13
A-2 2
5
A-2 6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
3
4
7
8
10
13
15
16
17
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
17
18
19

Problem 22 cont.

IV. F-1 6
7
F-2 1
7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 23.

I. A-1 1
4
5
A-2 1
12
A-3 1
4
C-1 1

II. A-1 1
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9

B-1 1
4
8
10
13
16

IV. A-1 1
A-2 1
6
A-3 1
6
B-1 1
3
6
9
12
13
D-1 4
D-2 7
E-1 1
3
4
5
17
18
19
F-1 7
F-2 5
7
8
G-1 12
G-2 5
19
H-1 10
H-2 8

Problem 24.

I. A-1 1
4
5
7
A-2 1
5
7
12
A-3 1
4
7

II. A-1 4
6
7
8
10
11
12
13
A-2 2
5
6
8
10
A-3 1
2
5
6
8
9
10
B-1 1
4
7
8
9
10
13
15
16
17

IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
D-1 4
11
18

E-1 1
3
4
5
6
17
19
F-1 7
F-2 7
8
G-1 3
10
12
G-2 5
17
19
21
H-1 10
H-2 6
8

Problem 25

I. A-1 1
4
5
7
A-2 1
5
6
A-3 1
4
6
C-1 1

II. A-1 1
6
7
8
1
1
1
A-2 2
6
7
8
9
1

Problem 25 cont.

II. A-3 1, 2, 3, 5, 6, 7, 9
B-1 7, 8, 13, 15
IV. A-1 1, 6
A-2 1, 6, 9
A-3 1, 6, 8, 11
B-1 1, 3, 6, 8, 9, 12
C-1 15
D-1 1, 2, 4
E-1 1, 3, 4, 5, 9, 16, 18
F-1 7
F-2 7, 8
G-1 15
G-2 3, 5, 22
H-1 7
H-2 10

Problem 26.

I. A-1 1, 9
A-3 1, 7, 9
C-1 1
II. A-1 8, 11, 13
A-2 6, 9, 10
A-3 6, 7
B-1 1, 4, 8, 17
IV. A-1 1
A-3 1
B-1 12
D-1 4
E-1 1, 4, 17
F-1 7
F-2 4, 8
G-1 10, 12
G-2 19, 23
H-2 6, 8, 15

Problem 27.

I. A-1 1, 4, 5, 8
A-2 1, 5, 7, 12
A-3 1, 4, 7
II. A-1 1, 4, 5, 6, 7, 8, 11, 13
A-2 2, 5, 6, 7, 8, 9, 10
A-3 1, 2, 3, 5, 6, 7, 8, 9, 10
B-1 1, 4, 7, 8, 10, 13, 15, 16, 17
IV. A-1 1, 6
A-2 1, 6, 10
A-3 1, 6, 10
B-1 1, 3, 6, 9, 12, 13
D-1 4, 11
E-1 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13
F-1 6, 7
F-2 3, 7, 8
G-1 10, 12
G-2 17, 19, 23
H-1 10
H-2 6, 8

Problem 28.

I. A-1 1, 4
A-2 1, 7, 12
A-3 1, 7
C-1 1
II. A-1 1, 2, 5, 6, 7

Problem 28 cont.

II. A-1 10
11
12
13
A-2 2
7
8
9
10
A-3 1
2
3
5
6
7
9
B-1 1
13
IV. A-1 1
A-2 1
11
A-3 1
10
B-1 1
3
6
9
12
D-1 2
E-1 3
4
5
18
F-2 7
8

Problem 29.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7
C-1 1
II. A-1 1
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
11
A-3 1
6
10
B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
17
18
19
F-1 7
F-2 7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 30.

I. A-1 1
A-2 1
12
A-3 1
II. A-1 2
7
8
10
11
12
13
A-2 8
10
A-3 1
2
5
6
7
IV. A-1 1
A-2 1
A-3 1
6
12

B-1 1
3
6
7
8
9
C-1 3
5
D-1 1
4
E-1 3
4
5
18
F-1 7
F-2 7
8
G-2 3

Problem 31.

I. A-1 1
4
8
A-2 1
7
12
A-3 1
7
C-1 1
II. A-1 1
6
7
8
10
11
13
A-2 2
6
7
8
9
10
A-3 1
2
3
5
6
7
9

Problem 31 cont.

II. B-1 1, 4, 8, 13, 15
IV. A-1 1
A-2 1, 10
A-3 1, 10
B-1 1, 3, 6, 9, 12
C-1 3
D-1 4
E-1 1, 3, 4, 17, 18
F-1 7
F-2 1, 2, 7, 8
G-1 12
G-2 3, 5, 23
H-2 8

Problem 32.

I. A-1 1, 4, 5, 8
A-2 1, 5, 7, 12
A-3 1, 4, 7
C-1 1
II. A-1 1, 6, 7, 8, 9, 10, 11, 12, 13
A-2 2, 5, 6, 7, 8, 9, 10
A-3 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13
B-1 1, 3, 4, 7, 8, 10, 13, 15, 16, 17
IV. A-1 1, 6
A-2 1, 4, 10
A-3 1, 6, 10
B-1 1, 3, 6, 9, 12, 13
D-1 4, 11
E-1 1, 3, 4, 5, 6, 17, 18, 19
F-1 6, 7
F-2 4, 7, 8
G-1 3, 10, 12
G-2 17, 19, 23
H-1 10
H-2 6, 8

Problem 33.

I. A-1 1, 4, 5, 8
A-2 1, 5, 7, 12
A-3 1, 4, 7
C-1 1
II. A-1 1, 4, 5, 6, 7, 8, 10, 11, 12, 13
A-2 2, 5, 6, 7, 8, 9, 10
A-3 1, 2, 3, 5, 6, 7, 8, 9, 10, 12, 13, 15, 16, 17
B-1 1, 3, 6, 10, 13
C-1 3, 12
D-1 4, 11
D-2 9
E-1 1, 3, 4, 5, 6, 13, 16
IV. A-1 1, 6
A-2 1, 6, 9, 10
A-3 1, 6, 10
B-1 4, 8, 10, 13, 15, 17

Problem 33 cont.

IV. E-1 17
18
19
F-1 7
F-2 4
7
8
G-1 10
12
G-2 3
4
5
19
23
H-1 10
H-2 2
6
8

Problem 34.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7

II. A-1 1
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10

A-3 1
2
3
5
6
7
8
9
10
B-1 1
2
3
4
7
8
10
13
15
16
17

IV. A-1 1
6
10
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
13
17
18
19
F-1 7
F-2 7
8
G-1 10
12
G-2 17
19
23
H-1 10
H-2 8

Problem 35.

I. A-1 1
4
A-2 1
A-3 1
4
9

II. A-1 6
7
8
10
11
13
A-2 2
6
8
9
10
A-3 1
2
5
6
7
9
B-1 8
10

IV. A-1 1
A-2 1
A-3 1
6
8
14
B-1 1
3
7
8
9
12
16
C-1 1
3
15
D-1 1
E-1 1
2
3
4
9
F-1 7
F-2 7
8
G-1 12
G-2 3
19
H-2 8

Problem 36.

I. A-1 1
A-2 1
7
12
A-3 1
7
C-1 1

II. A-1 8
11
12
A-2 6
9
A-3 6
B-1 7
8
16
17

IV. A-1 1
A-2 1
4
A-3 1
B-1 1
3
12
C-1 10
D-1 4
11
E-1 1
4
6
17

Problem 36 cont.
IV. F-1 7
F-2 7
8
G-1 10
G-2 19
23
H-2 6
8

Problem 37.
I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7
C-1 1
II. A-1 1
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
4
7
8
10
13
15
16
17
IV. A-1 1
6
A-2 1
4
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
17
18
19
F-1 6
7
F-2 7
8
G-1 3
10
G-2 17
19
23
H-1 10
H-2 6
8

Problem 38.
I. A-1 1
4
5
A-2 1
5
7
12
A-3 1
4
6
7
B-1 12
C-1 1
II. A-1 1
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
7
8
10
13
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
D-2 4
E-1 1
3
4
5
17
18
19
F-1 7
F-2 7
8
G-1 12
H-1 10
H-2 8

Problem 39.
I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7
II. A-1 1
4
6
7
8
10
11
12
13

Problem 39 cont.

II. A-2 2
4
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
4
7
8
10
13
14
15
16
17
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
E-1 17
18
19
F-1 7
F-2 4
7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 8

Problem 40.

I. A-1 1
4
A-2 1
A-3 1
C-1 1
II. A-1 4
7
8
11
12
13
A-2 2
6
9
10
A-3 2
6
7
8
9
B-1 1
4
8
10
13
15
16
IV. A-1 1
6
A-2 1
6
A-3 1
6
10
B-1 1
6
8
10
C-1 1
3
5
10
16
18
D-1 4
17
D-2 8
E-1 1
4
5
6
17
19
F-1 6
7
F-2 7
8
G-1 3
10
12
G-2 3
19
23
H-1 10
H-2 2
6
8
10

Problem 41.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 4
7
II. A-1 1
4
6
7
8
10
11
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
3
4
7
8
10
13
15
16
17
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10

Problem 41 cont.

IV. B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
17
18
19
F-1 6
7
F-2 7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 42.

I. A-1 1
4
A-2 1
12
A-3 1
II. A-1 1
6
7
10
11
13
15
A-2 2
7
8
9
10
A-3 1
2
3
5
6
7
8
B-1 13
IV. A-1 1
A-2 1
A-3 1
B-1 1
3
8
9
12
D-1 4
E-1 3
4
17
18
F-1 7
F-2 7
8

Problem 43.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7
II. A-1 1
4
6
7
8
10
11
13
A-2 2
5
6
A-2 7
8
9
10
A-3 1
2
3
5
6
7
8
9
12
13
B-1 1
4
7
8
10
13
15
16
17
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
7
8
9
12
13
D-1 4
11
E-1 1
3
4
5
6
13
17
18
19
F-1 7
F-2 7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 44.

I. A-1 1
4
5
A-2 1
5
12
A-3 1
II. A-1 1
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
3
4
8

Problem 44 cont.

II. B-1 13
15
16
IV. A-1 1
6
A-2 1
6
A-3 1
6
8
B-1 1
3
6
9
12
13
D-1 4
E-1 1
3
4
5
17
18
19
F-1 7
F-2 4
7
8
G-2 5
H-1 10

Problem 45.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7
II. A-1 1
4
6
7
8
A-1 10
11
12
13
15
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
4
7
8
10
13
15
17
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
C-1 3
D-1 4
11
E-1 1
3
4
E-1 5
6
17
18
19
F-1 7
F-2 7
8
G-1 12
G-2 5
17
19
23
H-1 10
H-2 2
6
8

Problem 46.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7
C-1 1
II. A-1 1
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 3
8
13
15
17
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
C-1 5
D-1 4
10
E-1 1
3
4
17
18
19
F-1 7
F-2 7
8
G-2 5
H-1 10

Problem 47.

I. A-1 1
4
5
8

Problem 47 cont.

I. A-2 1
5
7
12
A-3 1
4
7

II. A-1 1
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
4
7
8
10
13
15
16
17

IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10

B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
17
18
19
F-1 7
F-2 7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 48.

I. A-1 1
A-2 1
5
A-3 1
4
6
C-1 1

II. A-1 1
5
6
7
8
9
10
11
12
13
A-2 2
4
5
6
7
8
9
B-1 7
10
13
15

IV. A-1 1
A-2 1
6
9
10
A-3 1
8
B-1 1
3
6
7
8
9
12
13
D-1 4
E-1 1
3
4
5
6
16
17
18
19
F-1 7
F-2 1
2
7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 49.

I. A-1 1
4
8
A-2 1
7
12
A-3 1
7
C-1 1

II. A-1 1
6
7
10
11
13
A-2 2
7
8
9
10
A-3 1
2
3
5
6
7
9
B-1 1
8
17

IV. A-1 1
A-2 1
10

Problem 49 cont.
IV. A-3 1
B-1 1
3
9
D-1 4
E-1 3
4
18
F-1 7
F-2 7
8

Problem 50.
I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
6
7
II. A-1 1
4
6
7
8
10
11
12
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8

A-3 9
B-1 1
4
7
8
10
13
15
17
IV. A-1 1
6
A-2 1
4
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
C-1 3
5
10
D-1 4
9
11
E-1 1
3
4
5
6
17
18
19
F-1 6
7
F-2 7
8
G-1 10
12
G-2 3
5
17

G-2 19
23
H-1 10
H-2 6
8

Problem 51.
I. A-1 1
5
8
A-2 1
5
7
12
A-3 1
4
7
C-1 1
II. A-1 1
5
6
7
8
10
11
12
13
A-2 2
3
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
4
7
8
10

B-1 13
15
16
17
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
8
10
B-1 1
3
6
9
12
13
C-1 3
D-1 4
E-1 1
3
4
5
17
18
19
F-1 7
F-2 1
7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8

Problem 52.

I. A-1 1
4
A-2 1
12
A-3 1
II. A-1 1
5
6
7
10
11
12
13
15
A-2 2
7
8
9
10
A-3 1
2
3
5
6
7
9
B-1 3
8
IV. A-1 1
A-2 1
A-3 1
8
B-1 9
12
D-1 10
E-1 3
4
18
F-1 7
F-2 7
8

Problem 53.

I. A-1 1
4
5
8
A-2 1
5
7
12
A-3 1
4
7
II. A-1 1
6
7
8
10
11
13
A-2 2
5
6
7
8
9
10
A-3 1
2
3
5
6
7
8
9
B-1 1
3
4
7
8
10
13
15
16
17
IV. A-1 1
6
A-2 1
6
10
A-3 1
6
10
B-1 1
3
6
9
12
13
D-1 4
11
E-1 1
3
4
5
6
17
18
19
F-1 6
7
F-2 7
8
G-1 10
12
G-2 5
17
19
23
H-1 10
H-2 6
8